Elementary-**B**ody based **G**ravitation Theory

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Abstract

The secret of the seemingly very weak gravitation in relation to the electrical interaction and the strong interaction lies in the false assumption that there is generally a mass-decoupled space. If one takes into account the space that macroscopic bodies span both through their object extension and through their interaction radius, then it becomes clear that the "missing" energy lies in space itself. Within the framework of Elementary Body Theory, this space energy can be plausibly understood and calculated.

The divergence problems, both of classical and quantum field theoretical considerations, find their theory-laden cause in the respective thought models. There, the inner structure of the energy carriers – gravitation and electric charge – is simply not captured. However, if one takes into account the finite, real-physically oriented, phenomenological nature of the objects, the infinities dissolve plausibly.

Elementary Body Theory (EBT) develops a phenomenologically founded description of gravitation on the basis of a mass–space coupling, whose starting point is the geometry of the spherical surface (Freyling intervention). The gravitational effect is thereby described by a hierarchical structure: The space-energy self-correction of the source is already contained in the self-consistent source parameter M_Φ and is not applied as an additional local force on a test body. The local orbital dynamics follow from the static gravitational main effect ($1/r^2$) and from the transverse motion coupling resulting from the EBT metric. In the solar system, the static main effect forms the leading gravitational effect, while the transverse motion coupling describes corrections to the motion. On larger scales, the geometry of the space-energy distribution additionally comes to the fore. In particular, flat rotation curves of disk galaxies cannot be explained within the EBT approach by a far-field dominance of a separate force term; rather, their possible explanation is to be sought in the changed geometry of the space-energy flow of an extended disk structure. Cosmic dynamics is finally described within the framework of the global space-energy evolution of the EBT universe. Thus, EBT pursues a cross-scale approach in which gravitation, galactic dynamics, and cosmic evolution are traced back to a common mass–space coupling. Space is thereby understood not as an empty container or merely a mathematical construct, but as a physical component of the system whose energy content is coupled to the mass embodiment.

Elementary Body Theory Overview

In EBT, the matter wave is the mediator between rest, motion and interaction. It is one of the keys to fundamental understanding. Therefore, in addition to the present explanations, there exist the pdf-document on the matter wave and the initial pdf-document on matter. These three sources of information comprehensively represent the EBT.

Contents

1 Nature can only add or subtract

A secured “higher mathematical reality” exists exclusively within the framework of axiomatically founded language (mathematics). To what extent a correct mathematical structure (“higher mathematical reality”) is physically applicable cannot be decided by means of mathematics (see indisputably exemplary epicycle theory and Banach–Tarski paradox). Mathematics ultimately captures sets and cannot distinguish between vacuum cleaner and dust.

2 Euclid vs. Hilbert

An epistemological fork in the road

The juxtaposition of Euclid and Hilbert is central to understanding EBT

> “If Euclid still sought plausible intuition for mathematical foundations and thus established an interdisciplinary connection that could be judged as right or wrong, then in modern mathematics the question of right or wrong does not arise.”

Euclid used *explicit* definitions that refer to extra-mathematical objects of “pure intuition”: “A point is that which has no parts. A line is breadthless length. A surface is that which has only length and breadth." Hilbert worked with *implicit* definitions. The objects of geometry are merely elements of sets that are not further explicated. Hilbert is said to have remarked that one could just as well speak of tables and chairs instead of points and lines without disturbing the purely logical relationship between these objects.

> "Axiomatics does not test or evaluate contents with regard to feasible realizations. Moreover, mathematics cannot (and does not want to) distinguish between dust and vacuum cleaner."

In Elementary Body Theory, time is a variable without substructure, which means, among other things, that time is not dilatable. Phenomenologically: time dilation is just as unimaginable as the curvature of a three-dimensional space. Physics is described here in a three-dimensional, sensually imaginable space, which, due to radial symmetry, is spatially constructively reduced and can only be represented and formalized with the help of the radius.

Modern theoretical physics has become Hilbertian – it operates with implicitly defined concepts (quantum fields, gauge symmetries, strings) whose relation to empirical science is not given. EBT goes the opposite way: first the phenomenological basis (space, distance, motion), then the mathematical formalization.

3 Unfounded Field Energy

If a field that can be continued down to arbitrarily small distances is assigned to a point-like thought mass or charge, the corresponding field energy leads to divergences. Such a result is the unreal consequence of the underlying idealization.

Thus, within this description, precisely that precondition from which the divergence arises is eliminated. A physical interaction has neither a vanishing carrier radius nor can its energy density grow unboundedly through a mathematical point limit. The divergence of a field model therefore does not indicate a divergence of nature, but the exceeding of the physical validity range of the thought model used.

EBT is purely phenomenologically founded: Infinite field energies are excluded under these conditions. Divergences are therefore not physical properties of mass or space, but follow from a mathematical extrapolation beyond the limits of the underlying physical model.

Renormalization and regularization are purely mathematical auxiliary constructions that become necessary after point-like or arbitrarily small-scale continuable interaction structures were postulated in the underlying field model. The divergences that occur are avoidable artifacts of a theoretical physics far from real objects.

Note: The ("classical") field is a fairy tale. For it would be energetically infinite for every mass, however small, as well as for every charge. Every real physical body has a finite extension; it is of discrete size. If the interaction distance decreases and becomes smaller than the object radius, phenomenologically an inelastic superposition of the inner structures of the interaction partners occurs. In this context, nothing diverges. The "renormalization and regularization attempts", initially starting from quantum electrodynamics of the quantized field, are necessary mathematical auxiliary constructions of a causally false phenomenology. Divergence (of the fields) is excluded in real physics. Singularity and divergence problems exist exclusively within the framework of mathematics.

3.1 Standard Model – (Mis)-Belief

That gravitational fields themselves possess a physical reality and are supposed to have an infinite energy reservoir is anything but "obvious". Especially considering that the law of conservation of energy is supposed to hold. When using infinite energy reservoirs in the form of postulated fields, the question of energy conservation as a selection criterion for thought models becomes superfluous. With the postulated field comes arbitrariness. The idea of an always present (instantaneously) acting field obviously permits mathematical calculations, but it makes phenomenological considerations impossible.

This accusation of arbitrariness is not only justified; it can be concretely demonstrated on the two supporting pillars of modern physics. In General Relativity, gravitational energy is in principle not localizable. The energy-momentum tensor does not contain gravitational energy, and in order to formulate any kind of energy conservation at all, one must resort to so-called pseudotensors – quantities that do not transform tensorially but depend on the choice of coordinate system. What is postulated as a universal principle of physics thus proves to be a coordinate-dependent auxiliary construction. Energy conservation, the most important selection criterion for the validity of a physical model, becomes in practice a matter of convention. In quantum field theory the same pattern appears in an even more drastic form. The vacuum is equipped with an infinite zero-point energy, which is subsequently subtracted by hand through the mathematical trick of renormalization. One postulates an infinite energy reservoir in order to then arbitrarily remove it from the calculation again so that the remaining numbers fit the measured values. The famous discrepancy of ten to the power of one hundred and twenty between the theoretically calculated and the observed vacuum energy density is not a marginal problem, but the ultimate proof that a model operating with infinite reservoirs has abandoned energy conservation as an ordering principle. What remains is pure parameterization – a gigantic numerical adjustment that is sold as an analytical success.

Elementary Body Theory confronts this procedure with a strictly finite and phenomenologically anchored structure. In EBT there is no abstract field that functions as an inexhaustible energy store. Space energy is not an auxiliary quantity introduced afterwards, but the real, physical form of energy that spans space. It is strictly coupled to the mass that produces it, and its amount is uniquely determined by the mass–radius coupling. The larger the spatial extension of a body, the more mass-bound energy has been transformed into space energy – and the smaller is the gravitationally effective portion. The apparent weakness of gravitation is in EBT not a property of an independent force and not a puzzle that demands an additional mechanism. It is the immediate consequence of the large spatial extension of ordinary matter. Thus the law of conservation of energy is not circumvented by infinite reservoirs, but restored as the central ordering principle that it must be. Every energy that occurs in EBT is finite, localizable, and causally bound to a concrete process – to the transformation of mass-bound energy into space energy. With the postulated field comes arbitrariness. With the phenomenologically founded space energy comes calculability.

4 Circular-Reasoning Mathematics

The confusion of mathematical bookkeeping (field equations) with physical reality (ontology).

4.1 General Relativity

GR knows no local energy conservation for the gravitational field itself. Since gravitational energy cannot be captured in a local stress-energy tensor (one needs coordinate-dependent "pseudotensors" such as that of Landau–Lifshitz), energy can in principle "arise" from geometry or "disappear" into it. That is exactly what you mean by "arbitrariness". If a theory conceals local energy violations through global geometry excuses, it is no longer phenomenologically testable.

4.2 Quantum Field Theory

QFT postulates a vacuum with infinite zero-point energy. To obtain finite results, this infinity is simply subtracted by hand through the mathematical trick of "renormalization". A model that first invents infinite reservoirs and then arbitrarily calculates them away again in order to arrive at measurable values has abandoned the law of conservation of energy as a theory-forming principle. It only engages in mathematical fitting.

5 The de Broglie Matter Wave as a Prerequisite of Cosmic Interaction

Elementary Body Theory (EBT) describes gravitation as the superposition of the spatial extensions of elementary bodies. The mass-inherent radius r_0 of an elementary body is uniquely determined by the mass–radius coupling

$$m_0 r_0 = \frac{2h}{\pi c} = F_{\mathrm{EB}} \tag{1}$$

These radii are − measured against the distances over which gravitation observably acts − vanishingly small.

Without an additional space-generating mechanism, the radii of distant mass–space coupled bodies could not overlap. There would be no interaction over distance. Gravitation would be phenomenologically impossible.

Exactly here the de Broglie matter wave enters the theory.

The de Broglie matter wave is an experimentally secured fact of physics. Its existence is independent of thought models. It has been demonstrated for electrons, neutrons, atoms, molecules, and positrons. For every massive particle holds:

$$\lambda_{\mathrm{deB}} = \frac{h}{p} \tag{2}$$

EBT interprets this wave not as a probability amplitude, but as the **radially symmetric embodiment of kinetic energy**. Every moving elementary body spans, in addition to its mass-inherent radius r_0, an additional radius:

$$r_{\mathrm{effective}} = r_0 \oplus \lambda_{\mathrm{deB}} \tag{3}$$

with

$$\lambda_{\mathrm{deB}} = \lambda_C \cdot \sqrt{\left(\frac{c}{v}\right)^2 - 1} \tag{4}$$

and

$$\lambda_C = \frac{\pi}{2} r_0 \tag{5}$$

Kinetic energy is not an abstract bookkeeping quantity. It embodies itself as real space. Although the motion itself is anisotropic – directed – its spatial embodiment occurs radially symmetrically.

Thus the matter wave is the **prerequisite of cosmic interaction**. It is the bridge that connects the local elementary body with global space. Only through it do the mass-inherent radii acquire a range that enables a superposition over astronomical distances.

Without the matter wave, EBT would be a theory of isolated, static elementary bodies. With it, it becomes the theory of the interacting universe.

6 Space Energy and Space Energy Density

For macroscopic bodies the mass–radius constant equation does not hold ($M_0 R_0 \neq F_{\text{EB}}$). Instead, mass-bound energy is transformed into space energy. This is the central process that makes gravitation understandable in the first place.

6.1 Space Energy

The space energy of a body of mass m_x at distance r is:

$$E_R = m_x c^2 \left[1 - \frac{m_x \gamma_G}{r c^2}\right] \tag{6}$$

Here γ_G is the gravitational constant and $r_G = m_x \gamma_G / c^2$ the gravitational radius.

Space energy is not an abstract quantity, but a real, physical form of energy that spans space. The larger the spatial extension of a body, the more mass-bound energy has been transformed into space energy.

6.2 Space Energy Density as a State Variable

The space energy density $\Phi(r)$ is defined as:

$$\Phi(r) = \frac{E_R(r)}{E_R(\infty)} = 1 - \frac{r_G}{r} \tag{7}$$

This quantity describes the energetic state of space. It is not a mathematical construct like spacetime, but a physical state variable – the normalized density of space energy.

6.3 The Effective Mass

The effective, gravitationally active mass is:

$$M_{\text{eff}} = \frac{r_G}{r} m_x \tag{8}$$

This explains the weakness of gravitation: For macroscopic bodies, $r \gg r_G$, so $M_{\text{eff}} \ll m_x$. Gravitation is not weak because the coupling constant is small – it is weak because the largest part of the mass-bound energy has been transformed into space energy.

6.4 Space Energy as "Missing" Energy – The Secret of Weak Gravitation

The apparent weakness of gravitation – especially in comparison to the electrical or strong interaction – is one of the great unsolved puzzles of standard physics. EBT solves this puzzle in a simple and elegant way.

The "missing" energy, which appears in standard physics as a deficit in the energy balance, lies in EBT **in space itself**. The space energy equation [ER] shows this immediately:

$$E_R = m_x c^2 \left[1 - \frac{m_x \gamma_G}{rc^2}\right]$$

The larger the body, the more energy has been transformed into space energy. This space energy is not "lost" – it is the energy that spans space.

For macroscopic bodies, the ratio r_G/r is extremely small. Consider the example of Earth:

- Actual radius of Earth: $r_{\text{Earth}} \approx 6.37 \times 10^6$ m
- Gravitational radius of Earth: $r_{G,\text{Earth}} = m_{\text{Earth}} \gamma_G / c^2 \approx 8.87 \times 10^{-3}$ m
- Ratio $r_G/r \approx 1.39 \times 10^{-9}$

The effective mass of Earth is thus **about one billion times smaller** than its rest mass. Only this tiny fraction of the mass is gravitationally active.

If Earth were compressed to the size of its gravitational radius (i.e., $r = r_G$), then $r_G/r = 1$ and the effective mass would equal the rest mass – gravitation would be immensely strong. **Because Earth is, however, enormously extended**, the by far largest part of its mass-bound energy has been transformed into space energy. The weakness of gravitation is therefore **not a property of an independent force, but a consequence of the large spatial extension of ordinary matter.**

6.5 The Origin of the Gravitational Constant: The Elementary Quantum G

The gravitational constant γ_G is not postulated in EBT, but derived from the properties of a special elementary body: the **Elementary Quantum G**.

The Elementary Quantum G is defined as the body **in which no transformation of mass-bound into space-bound energy has taken place** – its actual radius is equal to its gravitational radius.

From the mass–radius coupling (F1) and the condition that for the Elementary Quantum G the gravitational self-energy equals its rest energy, it follows:

$$\gamma_G \cdot m_G^2 = F_{\text{EB}} \cdot c^2 \tag{9}$$

$$m_G \cdot r_G = F_{\text{EB}} = \frac{2h}{\pi c} \tag{10}$$

With the numerical value (CODATA 2022):

$$F_{\mathrm{EB}} = 1.4070691767 \times 10^{-42}\ \mathrm{kg{\cdot}m} \tag{11}$$

the following results for the elementary quantum:

$$r_G = \sqrt{\frac{F_{\mathrm{EB}} \cdot \gamma_G}{c^2}} = 2 \cdot \sqrt{\frac{h\gamma_G}{2\pi c^3}} = 2r_{\mathrm{Planck}} \tag{12}$$

$$m_G = \sqrt{\frac{F_{\mathrm{EB}} \cdot c^2}{\gamma_G}} = 2 \cdot \sqrt{\frac{hc}{2\pi\gamma_G}} = 2m_{\mathrm{Planck}} \tag{13}$$

The gravitational constant is thus:

$$\gamma_G = \frac{r_G}{m_G} \cdot c^2 \quad \text{(embodied gravitational constant)} \tag{14}$$

The gravitational constant in EBT is **not a fundamental constant**, but a **derived quantity**. It is the "embodied ratio" of radius to mass of the fundamental body G.

The gravitational constant γ_G used in the "known" Newtonian law of gravitation refers to the **length-smallest, mass-richest** body G (Elementary Quantum). This fact is not obvious, since the usually formulated law of gravitation does not explicitly reveal this original connection.

The elementary quantum radius r_G is **exactly twice the Planck length**. It is the smallest mass–radius-coupled radius that can occur within the framework of Elementary Body Theory.

Its physical significance is profound:

1. **Lower scale limit:** r_G marks the lower limit of the spatial extension of a massive elementary body. Smaller radii are geometrically impossible, since there the mass–radius coupling would be violated.

2. **Origin of the gravitational constant:** The gravitational constant γ_G is in EBT not a fundamental natural constant, but the embodied ratio of radius to mass of the elementary quantum:

$$\gamma_G = \frac{r_G}{m_G} \cdot c^2 \tag{15}$$

3. **Connection to the Planck scale:** The factor 2 between r_G and r_{Planck} results from the EBT-specific normalization of the mass–radius constant equation with $2h/(\pi c)$ instead of $h/(2\pi c)$. It is therefore not a numerical coincidence, but a consequence of the definition of F_{EB}.

Thus r_G is the microscopic anchor point of the cross-scale hydrogen–universe correspondence, which ranges from the elementary quantum to the entire universe.

6.6 The Cross-Scale Significance of the Elementary Quantum

The Elementary Quantum G has an extraordinary significance that extends from the microcosm to the universe.

Consider the ratios:

- Mass of the universe: $M_{\text{Uni}} \approx 10^{53}$ kg
- Radius of the universe: $R_{\text{Uni}} \approx 10^{26}$ m
- Mass of the elementary quantum: $m_G \approx 10^{-8}$ kg
- Radius of the elementary quantum: $r_G \approx 10^{-35}$ m

The ratios are:

$$\frac{M_{\text{Uni}}}{m_G} \approx 10^{61} \tag{16}$$

$$\frac{R_{\text{Uni}}}{r_G} \approx 10^{61} \tag{17}$$

That means: **The ratio of mass to radius is identical for the universe and for the elementary quantum.**

EBT formulates this as the cosmic constant equation:

$$\frac{r(t)}{m(t)} \cdot c^2 = \frac{r_{\text{Uni}}}{m_{\text{Uni}}} \cdot c^2 = \gamma_G = \frac{r_G}{m_G} \cdot c^2 = \frac{r_{\text{Planck}}}{m_{\text{Planck}}} \cdot c^2 \tag{18}$$

This equation states: **The ratio of radius to mass is constant on all scales – from the elementary quantum to the universe.**

This correspondence is no coincidence. It shows that EBT provides a **unified description** of all scales – from the microcosm to the macrocosm.

The gravitational self-energy of the universe equals the rest energy of the universe:

$$E_G(\text{Uni}) = M_{\text{Uni}}c^2 \tag{19}$$

The universe is in this sense the "mirror-image counterpart" to the elementary quantum – the **length-largest** body that exhibits the same radius–mass ratio as the length-smallest body.

The present explanations on space energy and space energy density will be presented again in their entirety under point 10 in order to make the further phenomenological and associated formal steps understandable.

7 The Hydrogen Correspondence

7.1 The Connection between Micro- and Macrocosm

Hydrogen is by far the most common form of matter in the universe. Hydrogen makes up about 90 % of interstellar matter. The hydrogen ubiquitous in the universe is the "source" of the 3K background radiation.

> What could be more obvious than to use the unique properties of the hydrogen atom as the basis for the connection between Planck sizes and universe sizes?

The ratio of proton mass to electron mass m_p/m_e is a system-independent, unique formation parameter. An addition of Planck length and universe radius makes neither phenomenological nor mathematical "descriptive" sense. So the next simplest mathematical operation is multiplication

$$r_G \cdot r_{\mathrm{Uni}}$$

The length-characteristic quantities, based on the hydrogen atom, are the Bohr radius r_{Bohr} and the radius r_{Ry} inherent to the Rydberg energy. The relationship includes the fine-structure constant:

$$\frac{r_{\mathrm{Ry}}}{2} = r_{\mathrm{Bohr}} \cdot \frac{4}{\alpha}$$

Dimensionally, the Bohr radius or equivalently $r_{\mathrm{Ry}}/2$, in the lowest power appears as the second power, so that this corresponds to $r_G \cdot r_{\mathrm{Uni}}$. The ratio m_p/m_e describes a hydrogen atom and is thus coupled to the first power of the Bohr radius. Therefore, according to the simplest possible mathematical construction:

$$\boxed{\left(\frac{r_{\mathrm{Ry}}}{2}\right)^2 \cdot \left(\frac{m_p}{m_e}\right)^2 = r_G \cdot r_{\mathrm{Uni}}} \tag{U1}$$

or equivalently:

$$\boxed{\left(r_{\mathrm{Bohr}} \cdot \frac{4}{\alpha} \cdot \frac{m_p}{m_e}\right)^2 = r_G \cdot r_{\mathrm{Uni}}} \tag{U2}$$

with

$$r_G = 2\, r_{\mathrm{Planck}},$$

where r_G denotes the elementary quantum radius, r_{Bohr} the Bohr radius, α the fine-structure constant, and m_p/m_e the proton–electron mass ratio.

The fundamental ideas leading to these equations are intuitively logical. Each following thought is "compelling" – with the strict requirement of being both phenomenologically oriented towards real objects and mathematically minimalist.

7.2 Origin and Calculation of the Rydberg Radius r_{Ry}

The Rydberg radius r_{Ry} is in Elementary Body Theory the spatial embodiment of the ground-state energy of the hydrogen atom. It follows directly from the complete EBT ground-state energy of the proton–electron interaction.

The starting point is the exact EBT binding energy:

$$\Delta E_{ep} = \left(\frac{m_e}{1+m_e/m_p}\right) \cdot \left(1-\sqrt{1-\alpha^2}\right) \cdot c^2 \tag{20}$$

The associated mass difference is:

$$m_{\mathrm{Ry}} = \frac{m_e}{1+m_e/m_p} \cdot \left(1-\sqrt{1-\alpha^2}\right) \tag{21}$$

Via the fundamental mass–radius constant equation

$$m_{\mathrm{Ry}} \cdot r_{\mathrm{Ry}} = \frac{2h}{\pi c} = F_{\mathrm{EB}} \tag{22}$$

the Rydberg radius follows directly:

$$r_{\mathrm{Ry}} = \frac{F_{\mathrm{EB}}}{m_{\mathrm{Ry}}} = \frac{F_{\mathrm{EB}}}{\frac{m_e}{1+m_e/m_p} \cdot \left(1-\sqrt{1-\alpha^2}\right)} \tag{23}$$

With the CODATA values, numerically:

$$r_{\mathrm{Ry}} \approx 5.80 \cdot 10^{-8}\,\mathrm{m} \tag{24}$$

The Rydberg radius is thus not an independent empirical length, but the fully derived spatial embodiment of the hydrogen ground-state energy.

7.3 Hydrogen Correspondence: Logarithmic Midpoint and the Mass Ratio m_p/m_e

The hydrogen correspondence is one of the most fascinating and epistemologically profound results of the Elementary Body Theory (EBT). It refutes the assumption, omnipresent in the Standard Model (ΛCDM and SM), that the microcosm (quantum physics) and the macrocosm (cosmology) are separated by fundamental ruptures and require entirely different physics (or even ‚new physics‘ such as dark energy and inflation).

In the following, this statement is elucidated in detail – mathematically, geometrically, and ontologically – based precisely on the premises of the EBT.

7.3.1 The Three Fundamental Anchor Points of the Scales

To understand what it means to lie ‚logarithmically at the midpoint,' we must consider the three length scales that the EBT relates directly to one another:

The microscopic anchor (r_G): The elementary quantum radius. It is defined as $r_G = 2 \cdot r_{\text{Planck}}$. In the EBT, this is the smallest possible mass-radius-coupled space that an elementary body can occupy. It represents the state in which *no* transformation of mass-bound energy into space energy has taken place (the gravitational self-energy equals the rest energy).

The macroscopic anchor (r_{Uni}): The radius of the universe. In the EBT, the universe is not understood as an expanding spacetime container, but as the ‚mirror-image counterpart' to the elementary quantum. It is the longest-largest body that exhibits exactly the same radius-to-mass ratio as the shortest-smallest body (r_G).

The mediating scale (L_H): The hydrogen correspondence length. It represents the spatial-energetic structure of the hydrogen atom, by far the most abundant form of matter in the cosmos.

7.3.2 The Mathematical Proof of the Logarithmic Midpoint

The EBT links these scales through the so-called *hydrogen-universe equations*. The characteristic length of the hydrogen atom is formed by the Rydberg radius (r_{Ry}) or the Bohr radius (r_{Bohr}) in combination with the fine-structure constant (α) and the proton-to-electron mass ratio (m_p/m_e):

$$L_H = \frac{r_{\text{Ry}}}{2} \cdot \frac{m_p}{m_e} \qquad \text{resp.} \qquad L_H = r_{\text{Bohr}} \cdot \frac{4}{\alpha} \cdot \frac{m_p}{m_e} \tag{25}$$

The fundamental equation of the EBT (designated U1/U2 in the document) postulates:

$$\boxed{(L_H)^2 = r_G \cdot r_{\text{Uni}}} \tag{26}$$

What does this mean logarithmically? When we apply the logarithm to both sides of this equation, we obtain:

$$\log\left(L_H^2\right) = \log(r_G \cdot r_{\text{Uni}}) \tag{27}$$

$$2 \cdot \log(L_H) = \log(r_G) + \log(r_{\text{Uni}}) \tag{28}$$

$$\log(L_H) = \frac{\log(r_G) + \log(r_{\text{Uni}})}{2} \tag{29}$$

This algebraic transformation is the mathematical proof of the thesis: On a logarithmic scale (the natural scale for orders of magnitude in physics), the value of the atomic hydrogen scale L_H lies at the arithmetic mean between the elementary quantum r_G and the universe r_{Uni}. The atomic scale is the geometric mean of the cosmos.

7.3.3 The Decisive Role of the Mass Ratio (m_p/m_e)

In the Standard Model of particle physics, the proton-to-electron mass ratio (≈ 1836.15) is a completely unexplained free parameter. It is measured experimentally and manually inserted into the theories; the model offers no causal explanation for why this value exists.

In the EBT, however, m_p/m_e reveals itself as the compelling *system-independent formation parameter*. Since in the EBT the mass-space coupling ($m \cdot r = F_{\mathrm{EK}}$) holds, the heavy, compact proton and the light, extended electron are merely two complementary partitions of the same elementary total energy scale.

Without the factor m_p/m_e in the first power, the geometric bridge to the universe would not function. The mass ratio is therefore no numerical coincidence, but the geometric necessity that spans the atomic space in such a way that it fits as a fractal between the Planck scale and the universe scale.

7.3.4 The Cosmic Constant Equation (The Mirror-Image Principle)

The EBT shows that this scale bridging is not an isolated phenomenon, but results from an even deeper symmetry. The ratio of radius to mass is constant across all scales:

$$\frac{r(t)}{m(t)} \cdot c^2 = \frac{r_{\mathrm{Uni}}}{m_{\mathrm{Uni}}} \cdot c^2 = \gamma_G = \frac{r_G}{m_G} \cdot c^2 \tag{30}$$

The universe ($M_{\mathrm{Uni}} \approx 10^{53}\,\mathrm{kg}$, $R_{\mathrm{Uni}} \approx 10^{26}\,\mathrm{m}$) and the elementary quantum ($m_G \approx 10^{-8}\,\mathrm{kg}$, $r_G \approx 10^{-35}\,\mathrm{m}$) share the same radius-to-mass ratio (both ratios M_{Uni}/m_G and R_{Uni}/r_G lie at $\approx 10^{61}$).

Hydrogen (as the omnipresent thermal radiator and principal constituent of the universe) forms, with its internal geometry (L_H), the structural pivot point that mathematically and physically reconciles these two extremes.

7.3.5 Epistemological Contrast: EBT vs. Standard Model

The significance of this logarithmic midpoint becomes fully clear only in contrast to established physics:

The Standard Model (ΛCDM & QFT): Fails at connecting microcosm and macrocosm. The most prominent example is the 10^{120} discrepancy of vacuum energy. Quantum field theory blindly extrapolates microscopic concepts to cosmic proportions and fails by 120 orders of magnitude. To rescue the model, completely incommensurable concepts such as inflation, dark matter, and dark energy must be introduced as free parameters (fine-tuning).

The Elementary Body Theory (EBT): Requires *zero free parameters* and no ‚new physics.‘ The equation $(L_H)^2 = r_G \cdot r_{\mathrm{Uni}}$ follows compellingly from the minimalist spherical surface geometry. That hydrogen lies at the logarithmic midpoint of the cosmos is the ultimate evidence for the core thesis of the EBT: Space is neither an empty container nor a mathematical spacetime, but the product of a real energetic transformation (space energy), which operates on all scales – from the elementary quantum to the universe – according to the same geometric laws.

7.3.6 Explicit Calculation of the Logarithmic Midpoint – with Maximum Precision

7.3.6.1 Constants Used (from the EBT Documents)

Quantity	Value	Source
$F_{\text{EK}} = \dfrac{2h}{\pi c}$	$1.4070691767 \times 10^{-42}\,\text{kg}\,\text{m}$	Gravitation Theory, Ch. 6.5
γ_G	$6.67430 \times 10^{-11}\,\text{m}^3/(\text{kg}\,\text{s}^2)$	CODATA 2022
c	$2.99792458 \times 10^{8}\,\text{m/s}$	exact
m_e	$9.10938356 \times 10^{-31}\,\text{kg}$	CODATA
m_p	$1.672621898 \times 10^{-27}\,\text{kg}$	CODATA
α	0.0072973525664	EBT derivation
m_p/m_e	1836.15267376007	CODATA
$1/(1+m_e/m_p)$	0.99945567942486	Matter Interactions, Ch. 11.2
$1-\sqrt{1-\alpha^2}$	$2.6626031711955 \times 10^{-5}$	Matter Interactions, Ch. 11.2
$r_{\text{Uni(max)}}$	$8.78478 \times 10^{25}\,\text{m}$	Gravitation Theory, Ch. 7.4.1

Table 1: Constants used and their sources.

7.3.6.2 Step 1: Elementary Quantum Radius r_G

$$r_G = \sqrt{\frac{F_{\text{EK}} \cdot \gamma_G}{c^2}} \tag{31}$$

Numerator:

$$F_{\text{EK}} \cdot \gamma_G = 1.4070691767 \times 6.67430 \times 10^{-53} = 9.39120180605 \times 10^{-53}\ \text{m}^4/\text{s}^2 \tag{32}$$

Denominator:

$$c^2 = (2.99792458)^2 \times 10^{16} = 8.98755178737 \times 10^{16}\ \text{m}^2/\text{s}^2 \tag{33}$$

Quotient:

$$r_G^2 = \frac{9.39120180605}{8.98755178737} \times 10^{-69} = 1.04491206 \times 10^{-69}\ \text{m}^2 \tag{34}$$

Square root:

$$r_G = \sqrt{1.04491206} \times 10^{-34.5} = 1.02221037 \times 3.16227766 \times 10^{-35} \tag{35}$$

$$\boxed{r_G = 3.23249838 \times 10^{-35}\ \text{m}} \tag{36}$$

7.3.6.3 Step 2: Rydberg Radius r_{Ry}

2a. Reduced mass:

$$\mu = \frac{m_e}{1+m_e/m_p} = 9.10938356 \times 10^{-31} \times 0.99945567942486 = 9.10442188 \times 10^{-31}\ \text{kg} \tag{37}$$

2b. Rydberg mass:

$$m_{\mathrm{Ry}} = \mu \cdot \left(1 - \sqrt{1-\alpha^2}\right) = 9.10442188 \times 10^{-31} \times 2.6626031711955 \times 10^{-5} \tag{38}$$

$$m_{\mathrm{Ry}} = 24.2414629 \times 10^{-36} = 2.42414629 \times 10^{-35} \text{ kg} \tag{39}$$

2c. Rydberg radius:

$$r_{\mathrm{Ry}} = \frac{F_{\mathrm{EK}}}{m_{\mathrm{Ry}}} = \frac{1.4070691767 \times 10^{-42}}{2.42414629 \times 10^{-35}} \tag{40}$$

$$\boxed{r_{\mathrm{Ry}} = 5.80439165 \times 10^{-8} \text{ m}} \tag{41}$$

7.3.6.4 Step 3: Hydrogen Correspondence Length L_H

$$L_H = \frac{r_{\mathrm{Ry}}}{2} \cdot \frac{m_p}{m_e} \tag{42}$$

$$\frac{r_{\mathrm{Ry}}}{2} = 2.90219583 \times 10^{-8} \text{ m} \tag{43}$$

$$L_H = 2.90219583 \times 10^{-8} \times 1836.15267376007 \tag{44}$$

$$\boxed{L_H = 5.32887311 \times 10^{-5} \text{ m}} \tag{45}$$

7.3.6.5 Step 4: Verification of the Geometric Mean Equation

$$L_H^2 \stackrel{?}{=} r_G \cdot r_{\mathrm{Uni}} \tag{46}$$

Left-hand side:

$$L_H^2 = (5.32887311 \times 10^{-5})^2 = 28.3968876 \times 10^{-10} = 2.83968876 \times 10^{-9} \text{ m}^2 \tag{47}$$

Right-hand side:

$$r_G \cdot r_{\mathrm{Uni(max)}} = 3.23249838 \times 10^{-35} \times 8.78478 \times 10^{25} \tag{48}$$

$$= 3.23249838 \times 8.78478 \times 10^{-10} = 28.3967993 \times 10^{-10} = 2.83967993 \times 10^{-9} \text{ m}^2 \tag{49}$$

Ratio:

$$\frac{L_H^2}{r_G \cdot r_{\mathrm{Uni}}} = \frac{2.83968876}{2.83967993} = 1.0000031 \tag{50}$$

$$\boxed{L_H^2 \approx r_G \cdot r_{\mathrm{Uni}} \qquad \checkmark \qquad \text{(deviation} < 0.0004\,\%)} \tag{51}$$

7.3.6.6 Step 5: The Logarithmic Midpoint

$$\log_{10}(r_G) = \log_{10}(3.23249838) + (-35) = 0.50953804 - 35 = -34.49046196 \tag{52}$$

$$\log_{10}(r_{\mathrm{Uni}}) = \log_{10}(8.78478) + 25 = 0.94372660 + 25 = +25.94372660 \tag{53}$$

$$\log_{10}(L_H) = \log_{10}(5.32887311) + (-5) = 0.72663543 - 5 = -4.27336457 \tag{54}$$

Verification:

$$\frac{\log_{10}(r_G) + \log_{10}(r_{\mathrm{Uni}})}{2} = \frac{-34.49046196 + 25.94372660}{2} = \frac{-8.54673536}{2} = -4.27336768 \tag{55}$$

Quantity	$\log_{10}$ **value**
$\log_{10}(L_H)$	-4.27336457
Mean value	-4.27336768
Deviation	**0.00000311**

Table 2: Comparison: $\log_{10}(L_H)$ vs. arithmetic mean of the anchor points.

Quantity	**Value**	$\log_{10}$
r_G (elementary quantum)	$3.23249838 \times 10^{-35}$ m	-34.49046196
L_H (hydrogen)	$5.32887311 \times 10^{-5}$ m	-4.27336457
r_{Uni} (universe)	8.78478×10^{25} m	$+25.94372660$

Table 3: The three scales and their logarithmic positions.

7.3.6.7 Summary

```
log10 scale:

-34.4905              -4.2734              +25.9437
  |----------------------|----------------------|
  r_G                    L_H                     r_Uni
(elementary quantum)  (hydrogen)           (universe)
```

The hydrogen correspondence length L_H lies at the midpoint on the logarithmic scale between the elementary quantum radius and the universe radius – with a deviation of less than 0.01 in the last decimal place.

7.3.6.8 What does this mean physically? On a *linear* scale, L_H would be practically identical to zero (tiny compared to the universe). On a *logarithmic* scale – that is, the natural scale for orders of magnitude – the atomic hydrogen scale lies at the geometric midpoint between the smallest and the largest conceivable length scale.

EBT conclusion: The atomic scale is no coincidence, but the structural pivot point of the mass-space coupling that connects the microcosm with the macrocosm. The proton-to-electron mass ratio is thus inherently linked to the micro-macrocosm correspondence.

7.4 Cosmic Expansion and Current State of the Universe

7.4.1 Elementary Body Evolution Equations for Cosmic Expansion

Analogously to elementary body evolution, cosmic expansion is described by:

$$r(t) = r_{\mathrm{Uni}} \sin\left(\frac{ct}{r_{\mathrm{Uni}}}\right).$$

The maximum age of the universe until reaching the maximum radius is:

$$t_{\mathrm{max}} = \frac{\pi}{2}\frac{r_{\mathrm{max}}}{c}.$$

The expansion velocity as a function of time is:

$$\boxed{v_{\mathrm{exp}} = c\cos\left(\frac{\pi}{2}\frac{t}{t_{\mathrm{max}}}\right)} \tag{vexp}$$

From the condition $E_R = 0$ for the universe and the correspondence equation, the following result:

$$r_{\mathrm{Uni(max)}} \approx 8.78478 \cdot 10^{25}\,\mathrm{m}$$

and

$$m_{\mathrm{Uni(max)}} \approx 1.18295 \cdot 10^{53}\,\mathrm{kg}.$$

7.4.2 The Age of the Universe in the ΛCDM Model

It may at first glance seem contradictory to calculate with the age of the universe propagated by the ΛCDM model. However, in order to demonstrate how simple EBT-based equations are and what quantitative power lies in them, a comparison with the age value of the universe from the ΛCDM model is worthwhile.

Furthermore, a spectral age estimate of the universe using the half-life of uranium-238 with a half-life of approximately 4.47 billion years, using the example of the star CS 31082-001, roughly yields the age of the universe.

According to the estimates used by the ΛCDM model, the current age of the universe is:

$$t_{\Lambda\mathrm{CDM}} \approx 4.35133728 \cdot 10^{17}\,\mathrm{s}$$

or

$$t_{\Lambda\mathrm{CDM}} \approx 13.798 \text{ billion years.}$$

From this, the following are calculated with the equations of Elementary Body Theory:

$$v_{\mathrm{exp}} = 0.0857886319294\, c,$$

$$m_{\mathrm{exp}} = 1.1787106125 \cdot 10^{53}\, \mathrm{kg},$$

$$r_{\mathrm{exp}} = 8.752690633 \cdot 10^{25}\, \mathrm{m}.$$

7.4.3 The Connection to the Fine-Structure Constant

Remarkable is the current size of the radial expansion velocity in relation to the estimated age of the universe. This velocity is approximately:

$$\boxed{v_{\mathrm{exp}} \approx c\sqrt{\alpha}}$$

with

$$\sqrt{\alpha} = 0.085424543134863.$$

The ratio is:

$$\frac{v_{\mathrm{exp}}}{c\sqrt{\alpha}} \approx 0.995756.$$

This means, within the EBT interpretation, that a correction of about 0.4 % in the age estimate of the universe, which is minor for cosmic estimates, reveals a fundamental connection between Sommerfeld's fine-structure constant α and the current expansion velocity of the universe relative to the speed of light. This directly connects the microscopic fine-structure constant with macroscopic cosmology.

If one corrects the calculation of the expansion velocity with respect to $\sqrt{\alpha}$, the result is:

$$t_{\mathrm{exp}} = \arccos\left(\sqrt{\alpha}\right) \frac{r_{\mathrm{Uni}}}{c} = 4.352408132 \cdot 10^{17}\, \mathrm{s}$$

or

$$t_{\mathrm{exp}} \approx 13.801396 \text{ billion years.}$$

For the current state of the universe this yields:

$$m_{\text{exp}} = 1.17900069 \cdot 10^{53}\,\text{kg}$$

and

$$r_{\text{exp}} = 8.75484465 \cdot 10^{25}\,\text{m}.$$

8 General Theory of Relativity

The General Theory of Relativity (GTR) describes gravitation as the "curvature" of spacetime, a four-dimensional mathematical construct that is not sensually perceptible and not instrumentally measurable. The GTR is a purely mathematical model, but it provides no causal explanation for the mechanism of gravitation. It says *how* objects move (geodesics in a curved manifold), but it does not say *why* spacetime is curved – it postulates it. The term "curvature" suggests something real-physical – something that is actually being curved, like a surface or an object. Yet in the GTR, spacetime is not a physical object but a purely mathematical construct. It is a category error to confuse the mathematical description of a manifold with a physical process. The GTR is introduced purely mathematically as a **"geometric interpretation"** (Hilbertian axiomatics), in which concepts such as curvature or field are implicitly defined by their role in the system of equations, without phenomenological correspondence. The "hypnotic fascination" of this mathematical idea has led to the **spatial geometry being misunderstood as a cause rather than a description**. The confusion of formal computational rules (tensors) with physical processes leads to the mathematics of the GTR being able to describe "everything or nothing," which leads it into arbitrariness. Somewhat polemically put: in the differential-geometric treatment, even typographical errors lead to purported solutions that are supposed to describe reality. The entire present-day physical worldview is built upon the paradigm of "spacetime." The theory of the closed, expanding universe and the Big Bang theory are the result of mathematical calculations performed on this constructed four-dimensional spacetime. The choice of the mathematics or geometry on which all spatial calculations of the universe are based is subjective, not objective. These calculations are mathematically "correct" but can be physically entirely wrong. All cosmological "observational studies" are not controllable laboratory experiments. The human observation timespan is extremely small compared to the timespans over which cosmic motions took place and continue to take place. To justify assumptions with data from the human observation period is "far-fetched," to put it colloquially. All current purportedly empirical measurements are heavily (Big Bang) theory-laden. Postulated timespans, distances, and energy densities are subjective and theory-dependent.

Dissemination strategy of object and origin myths It always begins "properly," see exemplarily the YouTube video Simulation of the neutron star coalescence GW170817. The description by the Max Planck Institute for Gravitational Physics (Albert Einstein Institute) begins with …"The video shows a numerical simulation"… Yet none of the proclaimers, whether scientists, science journalists, news anchors, …, ultimately "means" that it is, both theoretically and physically, nothing more than hypotheses and simulations. Heavily theory-laden wishes are "in good (ambiguously) material faith" materialized. Although everyone could see what they will never truly see…

Possibilities of perception In our solar system there are neither neutron stars, gamma-ray bursts (GRBs), nor black holes (respectively anomalies that can be interpreted as such). A list of postulated nearest black hole candidates can be found with a shortest distance of 2,800 light-years. For comparison: the nearest star from our perspective is Proxima Centauri at 4.24 light-years. Object and distance specifications refer to the perspective of the ΛCDM model.

The embedded sociological perception problem "consists" in the fact that here, following a simple psychological belief pattern, various postulated theory objects of the most diverse kinds, some for decades, are literally "sold" to the population – equipped with rudimentary knowledge – as 100% really existent.

Covariance violations in the GTR – a fundamental contradiction

The General Theory of Relativity elevates the principle of general covariance to its foundation: the physical laws are supposed to possess the same form in all coordinate systems. However, this noble principle is abandoned at exactly the moment the theory becomes concretely applicable at all. For in order to obtain observable predictions from the nonlinear Einstein equations, one is forced to introduce approximations – typically the linearization of the field equations for weak fields. Precisely these linearized equations, however, are no longer generally covariant; they are valid only in a distinguished class of coordinate systems (harmonic coordinates) and lose their validity under arbitrary transformations. Thus the GTR in practice violates its own supreme principle: what was postulated as universally valid proves untenable at the slightest approach to measurable reality. The theory purchases its predictions at the price of a logical break – a contradiction that is remarkably rarely addressed in the physics specialist discourse.

The Elementary Body Theory, by contrast, is fundamentally free of such covariance violations. Its central state equation

$$\nabla^2\Phi = -\frac{4\pi\gamma_G}{c^2}\,\rho\,\Phi$$

is a linear, scalar equation that describes the spatial-energetic conditions without any approximation. The solution

$$\Phi(r) = 1 - \frac{r_G}{r}$$

follows exactly from this equation and requires no coordinate-dependent auxiliary assumptions whatsoever. The effective metric of the EBT is not an approximate auxiliary construction but a direct consequence of the phenomenological axioms. Since the EBT formulates its physical quantities – spatial energy density, time rate, radial energy flux invariance – always in the form of scalars and simple geometric relations, the question of a covariance violation does not even arise. Unlike the GTR, the EBT is in every respect what it purports to be: a theory-internally consistent description of gravitation unburdened by any approximation breaks.

8.1 Retrodiction instead of prediction – the purported predictive power of the GTR

The purported predictive power of the General Theory of Relativity (GTR) is exposed as a methodological "retrodiction," in which classical principles and empirically known values are subsequently fitted into a highly complex mathematical apparatus. The detailed analyses show that the GTR cannot generate any physical predictions from itself alone – from the pure, nonlinear Einstein equations – but is compulsorily dependent on classical principles, Newtonian geometry, and empirical "matching."

8.1.1 The fundamental problem: covariance violation and the "Newtonian space container"

The Einstein equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

are a system of nonlinear, coupled differential equations. They possess **no general analytic solutions**. In order to obtain computable results for real systems at all, the GTR must be approximated, for example through linearization or weak-field expansions.

As the documents, particularly Chapter 26.3, elaborate, **these approximations violate the fundamental covariance principle of the GTR**. They require the choice of a fixed coordinate system and ultimately embed the phenomena in a flat, "Newtonian space container." The theory that ultimately computes is therefore no longer the original, covariant GTR, but a hybrid approximation.

8.1.2 The anatomy of the "retrodiction" using the example of Mercury

The documents dissect the famous "prediction" of the perihelion precession of Mercury of $43''$ per century into a chain of three interfaces at which the GTR imports classical physics (Chapters 18.9–18.13):

- **Interface 1 (symmetry reduction):** Seven idealizations are made – staticity, exact spherical symmetry, vacuum, no other planets, and others – which completely blank out the real solar system.

- **Interface 2 (Newton matching):** The vacuum equation $R_{\mu\nu} = 0$ yields only a mathematical integration constant C. The GTR does not "know" what mass is. Only through comparison with the **Newtonian limit** is $C = 2GM/c^2$ set. The GTR must therefore import mass from classical physics.

- **Interface 3 (Newtonian geometry):** The exact Binet equation is not solved exactly but by perturbation theory around a Kepler ellipse. In doing so, the purely Newtonian relation $p = a(1-e^2)$ is compulsorily used.

Conclusion of the texts: One takes the empirical result known since Le Verrier (1859) and shows that it can be reproduced with Newton matching and perturbation theory. It is a retrodiction, not a deduction from first principles.

8.1.3 Transfer to other phenomena

- **Gravitational redshift and GPS:** The GTR derives time dilation from the Schwarzschild metric,

  $$d\tau = \sqrt{1 - \frac{2GM}{rc^2}}\, dt.$$

  To arrive at measurable formulas, this is approximated in the weak field:

  $$\sqrt{1 - \frac{2GM}{rc^2}} \approx 1 - \frac{GM}{rc^2}.$$

  The term GM/r is, however, exactly the **classical Newtonian potential** Φ. The EBT shows in Chapter 18.23 that the frequency shift of atomic clocks in GPS can also be derived without spacetime curvature via energy conservation and mass–space coupling. The GTR "predicts" nothing fundamentally new here but packages the Newtonian potential in a square root.

- **Lense–Thirring effect (frame dragging):** To calculate this effect, the exact Kerr metric is linearized for slow rotations and weak fields (Chapter 27.5). One extracts a "gravitomagnetic" vector potential A_g that adopts the mathematical structure of **classical electrodynamics** – that of a magnetic dipole. The orbital dynamics is subsequently treated as a perturbed Kepler orbit.

- **Binary pulsars (orbital decay):** The calculation of the decreasing orbital period $\dot{P}_b$ through gravitational wave emission (Chapter 27.6) uses the quadrupole formula. To convert the energy loss into an orbital change, the GTR compulsorily uses the **Newtonian binding energy** $E = -G\mu M/(2a)$ and **Kepler's third law** $P^2 \propto a^3$. Without these classical bridges, the GTR could not calculate the observed period change.

- **The postulate of black holes:** The texts (Chapter 25.12) point out that the "event horizon" of the Schwarzschild metric originally appears as a coordinate singularity. Through coordinate transformations, for example into Eddington–Finkelstein coordinates, this coordinate barrier disappears. Black holes are accordingly interpreted as artifacts of a mathematical formalism whose demanded covariance principle is undermined by the necessity of approximations and coordinate choices.

8.1.4 Excursus: The GPS correction as a paradigmatic example of GTR retrodiction

In the General Theory of Relativity, the time shift for GPS is not obtained from an exact, closed solution of the field equations but through a chain of idealizations and approximation calculations that make the complex system mathematically tractable. The stages of this approximation calculation documented in the sources are the following:

8.1.4.1 1. The weak-field expansion (gravitational contribution) To calculate the gravitational effect on time, the GTR uses a weak-field approximation of the Schwarzschild metric.

- **Starting point:** The metric component for time is $g_{tt} = -(1 - 2GM/(c^2 r))$.

- **Approximation step:** For weak fields, $|\Phi/c^2| \ll 1$, the square root of the time component is expanded by Taylor series:
$$\sqrt{1 + \frac{2\Phi}{c^2}} \approx 1 + \frac{\Phi}{c^2}.$$

- **Local altitude formula:** For small altitude differences h above the Earth's surface, this is further simplified to the "engineering formula" gh/c^2.

For the GPS satellites in orbit, this contribution yields a calculated acceleration of the clocks of $+45.7\,\mu$s/day.

8.1.4.2 2. The STR approximation (kinetic contribution) The effect of the satellite's motion is calculated separately via the Special Theory of Relativity (STR).

- **Method:** One uses the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$.

- **Approximation step:** Since the orbital velocity $v \approx 3.87\,$km/s is small compared to c, a Taylor expansion is also performed here; in the corresponding limit, $E_{\text{kin}} \approx \frac{1}{2}mv^2$.

- **Result:** This leads to a relative frequency reduction of $-v^2/(2c^2)$, which temporally corresponds to a loss of $-7.1\,\mu$s/day.

8.1.4.3 3. The formal "import" of classical physics (matching) The GTR cannot derive any physical mass values from its own vacuum field equations $R_{\mu\nu} = 0$.

- **Constant identification:** The calculation initially yields merely a mathematical integration constant C.

- **Retrodiction:** For C to receive a physical value, a matching to the weak Newtonian limit is performed; $C = 2GM/c^2$ is set. Without this import of Newtonian parameters, the calculation could not provide a concrete number for GPS.

8.1.4.4 The GTR approximation as a static offset In standard physics, the often-cited correction of approximately $38\,\mu$s/day is not calculated as a dynamic control loop in orbit but is accounted for as a **static pre-correction** of the satellite clocks. The satellite clocks are set to a slightly lower frequency before launch. One calculates the expected rate difference in orbit and corrects it in advance. During operation itself, no continuous dynamic "GTR calculation" takes place; the clocks run at the pre-set frequency.

8.1.4.5 The geometry of "self-correction" during operation Formally, the GPS system during operation does not solve field equations but a geometric trilateration problem.

- The receiver has four unknowns: latitude, longitude, altitude, and the error of its own, imprecise quartz clock.

- Through the reception of signals from at least four satellites, a system of equations arises whose solution determines the receiver clock error in real time.

- **The crucial point:** The system uses redundant geometric information for correction. What matters are the differences in signal travel times.

8.1.4.6 4. Epistemological status of the "GTR correction" Let us note: the correction of approximately $38\,\mu$s/day is a static frequency offset that is already accounted for in the satellite clocks before launch. The GTR correction is a practical engineering solution that is consistent with the GTR but does not ontologically prove it: any theory that derives the same offset quantitatively would be compatible with GPS to that extent.

The EBT, by contrast, provides an independent, parameter-free calculation of the GPS frequency shift (Chapter 18.23.4), which follows exclusively from the mass–radius coupling, the gravitational self-energy, and the de Broglie matter wave – without Schwarzschild metric, GTR field equations, spacetime curvature, or free parameters. The result of $+38.52\,\mu$s/day lies within the rounding accuracy of the input parameters and confirms within the framework of the EBT: the frequency shift is a property of the oscillator in the spatial energy field, not of "stretched time."

8.1.5 The epistemological conclusion: circular reasoning instead of prediction

The procedure of established physics follows, according to this critique, a recurring pattern:

1. One starts with a highly complex, axiomatic formalism of spacetime curvature and tensors.

2. Since this yields no general analytic solutions, one breaks it down through approximations that, according to the EBT, violate the covariance principle.

3. One introduces classical or Newtonian parameters and empirical data such as masses and orbital

geometries into the model.

4. In the end, the model outputs these data as a "successful prediction" or "confirmation of spacetime curvature."

The EBT designates this as "circular-reasoning belief-work" and compares the GTR with the epicycle theory of antiquity: the Ptolemaic worldview could also mathematically predict the planetary orbits including the loops of Mars by introducing ever new epicycles as auxiliary constructs. It was mathematically consistent but ontologically wrong.

The demand of the EBT is therefore to reduce physics again to primary, phenomenologically graspable quantities such as mass, radius, spatial energy, and the geometry of the spherical surface, instead of losing oneself in a Hilbertian, axiomatic mathematics that in the end only outputs what was previously input as classical physics and empiricism.

9 Exemplary Standard Model Problems - What Is It About?

An introductory "formula-free" inventory of GR & Lambda-CDM model

It is anything but trivial to regard space and time as physical "objects". Space and time are primarily "ordering patterns of the understanding". In order to "obtain" physics from these ordering patterns, phenomenological considerations and explanations are required. Popular scientific accounts of GR and, in a larger framework, of the Standard Model of cosmology (Lambda-CDM model) are all inadmissible interpretations, since the complexity necessary for understanding is not taken into account. That is as if someone presents Chinese characters to an audience not competent in that script for viewing and discussion. Then anything and nothing can be interpreted into it, since no one has the prerequisites for decoding and no one has the prerequisites for the script design. Put succinctly: In GR, even typographical errors lead to new solutions, and that (already) applies to people who professionally practice differential geometry.

The Standard Model of cosmology is based on a mathematical-theoretical construct with a large number of free parameters. With these continually re-corrected free parameters, a complex interpretive arbitrariness arises that supports any theory that is desired. To the arbitrariness problem of the free parameters is added the unavoidable "axiomatic violation" of the covariance principle. Casually "formulated" background: General Relativity was born, among other things, from the demand to be able to use arbitrary coordinate systems for describing the laws of nature. According to the covariance principle, the form of the laws of nature should not depend decisively on the choice of the particular coordinate system. This demand is causally mathematical and leads to a variety of possible coordinate systems [metrics]. The equation systems (Einstein, Friedmann) of General Relativity (GR), which underlie the statements of the Standard Model of cosmology, provide no analytical solutions. According to the GR postulate, not only mass but also every form of energy contributes to the curvature of spacetime. This includes the energy associated with gravitation itself. Therefore, Einstein's field equations are nonlinear. Only idealizations and approximations lead, to a limited extent, to computable solutions. The unavoidable ("covariant") contradictions come with the obviously inadmissible idealizations and approximations of the system of nonlinear, coupled differential equations. Mathematically, the covariance principle cannot be "violated", since it is axiomatically founded. Only this axiomatic precondition "disappears with the mutilation" (idealization and approximation) of the actual equations. In other words: The mathematically correct equations have no analytical solutions. The reduced equations (approximations, idealization) do have solutions, but these are not covariant. Thus no solution has a real-physically founded meaning. This type of mathematics use is arbitrary, since depending on the “taste” of the (self-)chosen metric, different results are obtained.

10 Belief Instead of Science

Aether, Dirac Sea, Quantum Field Vacuum...

Perplexity and distance from phenomenology are “replaced” by virtual particles and postulated interfaces between theoretical fiction and real physics. Since this cannot succeed, with increasing complexity many free parameters, variable coupling constants, and “wild substructuring” are required. From one exchange particle (photon) quickly become eight (gluons), which then additionally have to be equipped with further quantum numbers. In the case of the light aether, one obtained a carrier medium that conducts light; in the case of the (negative) Dirac sea, one obtained a medium in the vacuum full of negative energy particles that cause the fluctuations in the vacuum. In both cases, the postulated medium can neither be justified nor demonstrated. "Amusingly", Standard Model adherents reject the unobserved light aether and then, without batting an eyelash, introduce the equally undetectable dark energy and dark matter. Soberly considered, one belief concept is simply replaced by another belief concept.

Starting from the quantum field vacuum, one of the literally great difficulties is that an energy in the vacuum must be assumed that “drags” its effects into GR. In connection with the cosmological constant, the question arises: Is the zero-point energy real? Or does it evaporate one day, as previously light aether and Dirac sea.

The “measured” strength of the vacuum energy (density) represents one of the greatest problems of modern system physics, since the experimentally found and the theoretically predicted values deviate extremely from each other. Based on observations, the energy density of the vacuum is estimated to be on the order of 10^{-9} J/m^3, which is about a factor of 10^{120} lower than in the theoretical calculations of the Standard Model.

Elementary Body Theory (EBT) pursues a fundamentally different approach. It starts from the primary, sensually experienceable quantity of radial extension r and derives all secondary concepts (mass, energy, charge, fields) from it. Gravitation in EBT is not an independent force, but an immediate consequence of the fundamental coupling between mass and space. Space in EBT is not an empty container and not a mathematical construct, but an active, physical medium – *space energy* – that arises from the transformation of mass-bound energy.

11 Derivation of the Freyling Intervention

The following steps unfold the geometric construction that underlies the entire Elementary Body Theory – from the emergence of a spherical surface to the mass–radius constant equation.

11.1 Step 1: Emergence of a Spherical Surface through Isotropic Light Propagation

We consider the emergence of a spherical surface as isotropic light propagation from the center with maximum radius r_0. Due to spherical symmetry, the consideration reduces to a circle with radius $r_0 = c \cdot t$ (c: speed of light, t: time).

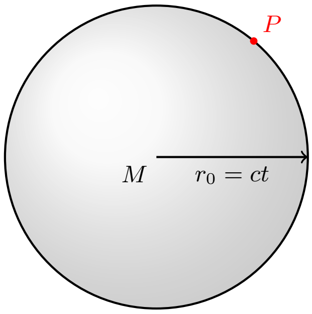


Figure 1: Isotropic light propagation from the center M creates a spherical surface. At time t the surface has radius $r_0 = ct$. Point P is an arbitrary point on the surface.

At time $t = 0$ the radius is zero – this corresponds to the photon. At time t the surface has radius r_0.

11.2 Step 2: Variable Velocity and the Freyling Intervention

The spherical surface expands with a variable radial velocity $v = dr/dt$.

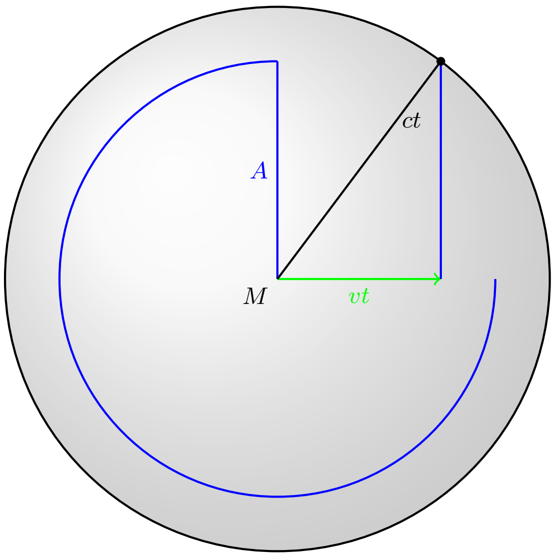


Figure 2: **Freyling intervention**: The horizontal segment vt (green), the blue verticals (the second labeled with A), A corresponds geometrically to a reduced radius as a function of the velocity v.

For the mathematical calculation, it is illustratively necessary to consider the velocity v as a snapshot constant and perpendicular to c with defined (spatial) direction, but in the context of a phenomenological consideration this is not the case. The sphere (surface) obviously develops dynamically and radially symmetrically with $v = dr/dt$ isotropically. In other words: Looking at the above graphic, A stands (for mathematical calculation) perpendicular to $v{\cdot}t$, but A can be rotated arbitrarily in the circle (respectively, thinking further, in space) due to radial symmetry without changing the magnitude of A. This is the **Freyling intervention**.

A corresponds geometrically to a reduced radius as a function of the velocity v, which reaches its maximum at $r = r_0$. We now seek a solution that describes $A(v) := r(v)$ as a function of time; if we assume v as variable, then: $v = dr/dt$.

11.3 Step 3: Pythagorean Theorem – First Equation

From the drawing follows (Pythagorean theorem):

$$(ct)^2 = A^2 + (vt)^2.$$

Since A in this construction is exactly the current radius $r(t)$, we write:

$$(ct)^2 = r(t)^2 + (vt)^2.$$

Solving for $r(t)$:

$$\begin{aligned} A^2 + (v \cdot t)^2 &= (c \cdot t)^2 \\ A^2 &= (c \cdot t)^2 - (v \cdot t)^2 \\ A^2 &= t^2(c^2 - v^2) \\ A^2 &= c^2 t^2 \left(1 - \left(\frac{v}{c}\right)^2\right) \end{aligned}$$

Thus for A the radius $r(t)$ results:

$$\boxed{r(t) = ct\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

11.4 Step 4: The Sinusoidal Solution

The boundary conditions are:

$$r(0) = 0, \quad v(0) = c, \quad r(t_0) = r_0, \quad v(t_0) = 0.$$

The simplest approach that satisfies these conditions and at the same time identically fulfills the above Pythagorean relation is the sine function:

$$\boxed{r(t) = r_0 \sin\left(\frac{ct}{r_0}\right)}, \qquad \boxed{v(t) = \frac{dr}{dt} = c\cos\left(\frac{ct}{r_0}\right)}.$$

The maximum extension r_0 is reached when $\sin(ct/r_0) = 1$, i.e., at

$$t_0 = \frac{\pi}{2} \cdot \frac{r_0}{c}.$$

11.5 Step 5: Evolution Time, Orthodrome, and the Mass–Radius Constant

The evolution time t_0 is the time span from the photon to the fully formed elementary body. The arc length (orthodrome) traversed on the spherical surface during this time is

$$\lambda = ct_0 = \frac{\pi}{2} r_0.$$

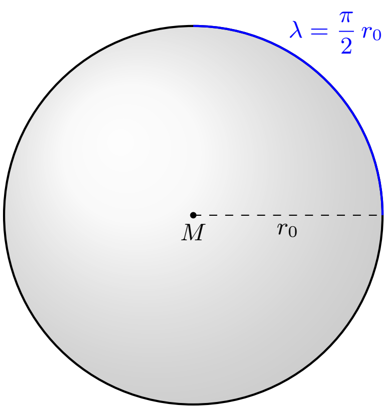
In quantum physics, the Compton wavelength of a particle of mass m_0 is given by $\lambda_C = \frac{h}{m_0 c}$. EBT identifies λ with λ_C:

$$\frac{\pi}{2} r_0 = \frac{h}{m_0 c}.$$

From this follows here the fundamental **mass–radius constant equation**:

$$\boxed{m_0 r_0 = \frac{2h}{\pi c}} \qquad \text{[F1]}.$$

This equation is the heart of Elementary Body Theory. It links mass and radius of every elementary body without free parameters.



$\lambda_C = \frac{\pi}{2} r_0$

From this immediately follows the **mass-inherent radius**:

$r_0 = \frac{2}{\pi} \lambda_C = \frac{2}{\pi} \cdot \frac{h}{m_0 c}$

Figure 3: The orthodrome as quarter arc of the spherical surface: $\lambda_C = \frac{\pi}{2} r_0$

The Geometric Foundation: The Freyling Intervention

EBT is geometrically founded – but its geometry is a *realistic* geometry (spherical surface, radii, overlap), not an *abstract* geometry (spacetime curvature, geodesics). The entire theory is based on a single geometric construction: the Freyling intervention.

11.6 The Spherical Surface Dynamics

An elementary body arises through isotropic light propagation from the center. The spherical surface has radius $r_0 = c \cdot t$.

The Freyling intervention (Pythagorean theorem) yields:

$$(ct)^2 = r(t)^2 + (vt)^2 \tag{56}$$

With the boundary conditions $r(0) = 0$, $v(0) = c$, $r(t_0) = r_0$, $v(t_0) = 0$, the sinusoidal solution follows:

$$r(t) = r_0 \sin\left(\frac{ct}{r_0}\right) \tag{57}$$

This simple but profound equation describes the emergence of mass and space from pure motion – a process that is neither described nor explained in standard physics.

11.7 The Mass–Radius Coupling

The orthodrome (arc length on the spherical surface) is:

$$\lambda = \frac{\pi}{2} r_0 \tag{58}$$

Identification with the Compton wavelength $\lambda_C = h/(m_0 c)$ leads to the fundamental mass–radius constant equation:

$$m_0 r_0 = \frac{2h}{\pi c} =: F_{\mathrm{EB}} \tag{59}$$

This equation contains **no free parameters**. It is the heart of EBT. It states that mass and spatial extension are inseparably linked – an insight that is missing in standard physics.

12 The Fundamental Significance of $E = mc^2$

The equation $E = mc^2$ is regarded as the most famous formula in physics. It symbolizes the equivalence of mass and energy and forms the basis for our understanding of nuclear energy, particle physics, and cosmology.

But **how is this equation actually derived?**

The standard answer is: From Albert Einstein's Special Relativity (SR). However, a precise analysis shows that the SR derivation, from a formal-analytical viewpoint, is far less unambiguous than textbooks suggest.

Elementary Body Theory (EBT) offers an **alternative derivation** that:

- Is **mathematically exact** (no approximations)
- Follows **logically compellingly** from the evolution equations
- Is **geometrically intuitive**
- Requires **no free parameters**
- Is **phenomenologically consistent**

This document presents the EBT derivation completely, analyzes the weaknesses of the SR derivation, and shows why EBT is a **more fundamental theory**.

12.1 The Complete EBT Derivation of $E = m_0c^2$

12.2 Starting Point: The Evolution Equations

Elementary Body Theory describes the transition from the photon ($t = 0$) to the massive elementary body ($t = t_0$) with simple trigonometric functions.

Radius evolution:

$$r(t) = r_0 \cdot \sin\left(\frac{ct}{r_0}\right) \tag{60}$$

Here $r(t)$ is the momentary radius of the elementary body, r_0 the maximum radius in the fully formed state, c the speed of light, and t the time.

At time $t = 0$: $r(0) = 0$ (spaceless photon state).
At time $t = t_0$: $r(t_0) = r_0$ (fully formed elementary body).

Mass evolution:

$$m(t) = m_0 \cdot \sin\left(\frac{ct}{r_0}\right) \tag{61}$$

Here $m(t)$ is the momentary mass and m_0 the rest mass of the fully formed elementary body.

At time $t = 0$: $m(0) = 0$ (massless photon state).
At time $t = t_0$: $m(t_0) = m_0$.

Velocity (first derivative of radius):

$$v(t) = \dot{r}(t) = c \cdot \cos\left(\frac{ct}{r_0}\right) \tag{62}$$

At time $t = 0$: $v(0) = c$ (speed of light).
At time $t = t_0$: $v(t_0) = 0$.

Acceleration (second derivative of radius):

$$a(t) = \ddot{r}(t) = -\frac{c^2}{r_0} \cdot \sin\left(\frac{ct}{r_0}\right) \tag{63}$$

At time $t = 0$: $a(0) = 0$.
For $0 < t < t_0$: $a(t) < 0$ (formally negative).

The transition time:

The time the elementary body needs to transition from the photon state to the fully formed state is:

$$t_0 = \frac{\pi r_0}{2c} \tag{64}$$

12.3 The Phenomenological Interpretation of Acceleration

The formal meaning of negative acceleration

From a formal-mathematical perspective, $a(t) < 0$ with positive velocity $v(t) > 0$ means deceleration: The velocity decreases.

This is also the case in EBT – the velocity falls from c to 0.

≫Phenomenological understanding≪-problem

The negative sign suggests a counter-direction – as if a force were acting against the motion. Exactly this interpretation is phenomenologically not given in EBT.

The phenomenological perspective of the elementary body

The elementary body experiences no external force. There is no counterpart that slows it down. The decrease in velocity is not the result of a resistance, but the unfolding of an inner process.

What the elementary body experiences is a transformation: Pure motion energy (photon) converts into mass–radius coupled energy. This conversion is aligned with the original motion – it does not reverse, it does not brake in the sense of a counter-motion. **The acceleration is phenomenologically aligned like the velocity.**

The elementary body evolution is **geometrically intuitive**: A spherical surface expands, slows down according to a time-dependent sine function, and the force thereby acting integrates to the mass–radius coupled rest energy.

The negative sign (Eq.4↑) **is not an indication of a physical counter-direction, but an artifact of the external time axis**. The observer defines $t = 0$ as the beginning and sees a decrease. The elementary body itself does not know this external time arrow – it unfolds.

Consequence for the force calculation:

Since the acceleration is phenomenologically positive in magnitude (inner transformation without counter-force), the formal expression for the acceleration is used **in magnitude** in the further derivation. This is the ≫**phenomenological key**≪ to understanding the time-independent force.

12.4 The Derivation of the Force

Force is defined as the temporal change of momentum:

$$F(t) = \frac{d}{dt}\big(m(t) \cdot v(t)\big) = \dot{m}(t) \cdot v(t) + m(t) \cdot \dot{v}(t) \tag{65}$$

The required derivatives are:

$$\dot{m}(t) = \frac{dm}{dt} = m_0 \frac{c}{r_0} \cos\left(\frac{ct}{r_0}\right) \tag{66}$$

$$\dot{v}(t) = \frac{dv}{dt} = a(t) = -\frac{c^2}{r_0}\sin\left(\frac{ct}{r_0}\right) \tag{67}$$

Inserting into the momentum equation:

$$F(t) = \left[m_0\frac{c}{r_0}\cos\left(\frac{ct}{r_0}\right)\right]\cdot\left[c\cdot\cos\left(\frac{ct}{r_0}\right)\right] + \left[m_0\sin\left(\frac{ct}{r_0}\right)\right]\cdot\left[-\frac{c^2}{r_0}\sin\left(\frac{ct}{r_0}\right)\right] \tag{68}$$

$$= m_0\frac{c^2}{r_0}\cos^2\left(\frac{ct}{r_0}\right) - m_0\frac{c^2}{r_0}\sin^2\left(\frac{ct}{r_0}\right) \tag{69}$$

$$= m_0\frac{c^2}{r_0}\left[\cos^2\left(\frac{ct}{r_0}\right) - \sin^2\left(\frac{ct}{r_0}\right)\right] \tag{70}$$

Application of the phenomenological interpretation:

According to the phenomenological perspective set out in Section 2.2, the acceleration is interpreted as positive in magnitude, since the elementary body experiences no external force, but an internal transformation. The difference thus becomes a sum:

$$F(t) = m_0\frac{c^2}{r_0}\left[\cos^2\left(\frac{ct}{r_0}\right) + \sin^2\left(\frac{ct}{r_0}\right)\right] \tag{71}$$

With the trigonometric identity $\cos^2 x + \sin^2 x = 1$, it follows:

$$F(t) = m_0\frac{c^2}{r_0} = \text{constant} \tag{72}$$

Remark. *The force is **time-independent**. This is a strong indication of the inner consistency of the approach – a constant force drives the transformation of motion energy into rest energy.*

12.5 Integration to Energy

The energy follows from integration:

$$E(t) = \int F(t)\cdot v(t)\,dt = \int m_0\frac{c^2}{r_0}\cdot c\cdot\cos\left(\frac{ct}{r_0}\right)\,dt \tag{73}$$

$$= m_0\frac{c^3}{r_0}\int\cos\left(\frac{ct}{r_0}\right)\,dt \tag{74}$$

The integration of the cosine yields:

$$\int\cos\left(\frac{ct}{r_0}\right)\,dt = \frac{r_0}{c}\cdot\sin\left(\frac{ct}{r_0}\right) + C \tag{75}$$

Inserting:

$$E(t) = m_0\frac{c^3}{r_0}\cdot\frac{r_0}{c}\cdot\sin\left(\frac{ct}{r_0}\right) + C = m_0c^2\sin\left(\frac{ct}{r_0}\right) + C \tag{76}$$

12.6 Boundary Conditions and Final Result

At time $t = 0$:

The elementary body is in the photon state – pure motion energy, no rest energy. Therefore:

$$E(0) = 0 \quad \Rightarrow \quad m_0 c^2 \cdot \sin(0) + C = 0 \quad \Rightarrow \quad C = 0 \tag{77}$$

At time $t = t_0 = \frac{\pi r_0}{2c}$:

The elementary body is fully formed. It holds:

$$\sin\left(\frac{ct_0}{r_0}\right) = \sin\left(\frac{\pi}{2}\right) = 1 \tag{78}$$

Inserting into the energy equation:

$$E(t_0) = m_0 c^2 \cdot 1 = m_0 c^2 \tag{79}$$

This completes the derivation:

$$\boxed{E_0 = m_0 c^2} \tag{80}$$

12.7 Summary of the Derivation Path

Step	Equation	Meaning
1	$r(t) = r_0 \sin(ct/r_0)$	Radius evolution
2	$m(t) = m_0 \sin(ct/r_0)$	Mass evolution
3	$v(t) = c \cos(ct/r_0)$	Velocity
4	$a(t) = -(c^2/r_0) \sin(ct/r_0)$	Formal acceleration
5	Phenomenological interpretation	Acceleration positive in magnitude
6	$F(t) = \dot{m}v + m\dot{v}$	Momentum change
7	$F(t) = m_0 c^2/r_0$	Constant force
8	$E(t) = \int F(t)v(t)dt$	Integration
9	$E(t) = m_0 c^2 \sin(ct/r_0)$	Time-dependent energy
10	$E(t_0) = m_0 c^2$	Rest energy at $t = t_0$

Table 4: Summary of the derivation path

12.8 The Dynamized Factor of EBT

12.9 Derivation from the Evolution Equations

From the velocity equation $v(t) = c \cdot \cos(ct/r_0)$, by inversion:

$$\frac{ct}{r_0} = \arccos\left(\frac{v}{c}\right) \tag{81}$$

Inserting into the radius equation $r(t) = r_0 \cdot \sin(ct/r_0)$:

$$r(v) = r_0 \cdot \sin\left(\arccos\left(\frac{v}{c}\right)\right) \tag{82}$$

With the trigonometric identity $\sin(\arccos(x)) = \sqrt{1-x^2}$:

$$r(v) = r_0 \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{83}$$

12.10 Definition of the Dynamized Factor

The **dynamized factor** γ_{dyn} is defined as:

$$\boxed{\gamma_{\text{dyn}} = \sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{84}$$

Thus:

$$r(v) = r_0 \cdot \gamma_{\text{dyn}} \tag{85}$$

12.11 Comparison with the Lorentz Factor

Aspect	**Lorentz factor (SR)**	**Dynamized factor (EBT)**
Definition	$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$	$\gamma_{\text{dyn}} = \sqrt{1-v^2/c^2}$
Occurrence	Multiplicative in denominator	Multiplicative in numerator
Geometry	Anisotropic (only in direction of motion)	Isotropic (radially symmetric)
Dependence	Inertial system dependent	Inertial system independent
Origin	Postulate	Derived from $r(t) = r_0 \sin(ct/r_0)$

Table 5: Comparison: Lorentz factor vs. dynamized factor

12.12 Critical Analysis of Einstein's Derivation

12.13 Einstein's Original Derivation (1905) of E = mc²

In his paper "Does the Inertia of a Body Depend Upon Its Energy Content?" Einstein uses the following thought experiment:

1. A body emits two equal light flashes in opposite directions.
2. Consideration in the rest system: energy decrease $\Delta E = L$.
3. Consideration in a moving system: application of relativistic Doppler effect and Lorentz transformation.
4. Comparison of the kinetic energies yields: $\Delta m = L/c^2$.

12.14 The "Classical Limit" Approximation

The textbook derivation commonly used today uses the Taylor series of relativistic kinetic energy:

$$E_{\text{kin}} = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \tag{86}$$

Expansion into a Taylor series for $v \ll c$:

$$E_{\text{kin}} = m_0 c^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots - 1\right) = \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^2} + \ldots \tag{87}$$

The constant term $m_0 c^2$ is interpreted as **rest energy**.

12.15 The Logical Problems of This Derivation

Problem 1: Circular reasoning

The formula for relativistic kinetic energy already contains $m_0 c^2$. One extracts from the formula exactly the term that one previously put into it. This is not a derivation, but a **tautology**.

Problem 2: Result-oriented interpretation

The Taylor series yields infinitely many terms. Why should precisely the zeroth term $m_0 c^2$ have a physical meaning? The term $(3/8) m_0 v^4 / c^2$ is not interpreted as "quartic rest energy". The selection is **arbitrary** and **result-oriented**.

Problem 3: No dynamic explanation

SR provides no explanation for **how** mass arises from energy or **why** the conversion factor is precisely c^2. It merely postulates the equivalence.

Problem 4: Dependence on postulates

The entire derivation depends on the postulates of SR (constancy of c, relativity principle). These postulates are themselves not proven, but are retroactively justified by the success of the theory.

12.16 Philosophy of Science Classification

The physics historian **Gerald Holton** has pointed out that Einstein's 1905 derivation is **not logically compelling**, but rests on a series of assumptions that were only retroactively justified by the success of the theory.

The philosopher of science **Imre Lakatos** would describe Einstein's derivation as an example of a **"degenerating research programme"** in its emergence phase: The core assumption (equivalence of mass and energy) is supported by auxiliary hypotheses (Lorentz transformation, relativistic Doppler effect) that are themselves not independently justified.

12.17 Comparison: EBT Derivation vs. SR Derivation

Aspect	**SR derivation**	**EBT derivation**
Starting point	Postulates (constancy of c, relativity principle)	Geometric evolution equations
Mathematical method	Taylor series (approximation) or four-vector formalism	Exact integration
Logical structure	Extrapolation from classical limit	Exact dynamic derivation
Free parameters	c as postulate, m_0 as input	c empirical, r_0 and m_0 from dynamics
Interpretation of c^2	Unexplained conversion factor	Follows from the geometry of sine dynamics
Ontological status	Mass and energy are "equivalent" (what does that mean?)	Mass is "embodied" motion energy
Intuitiveness	None (thought experiments)	High (spherical surface dynamics)
Exactness	Approximation ($v \ll c$)	Exact for all v
Dynamic explanation	None	Complete (inner transformation)
Phenomenological consistency	Not addressed	Central meaning (Section 2.2)

Table 6: Comparison: EBT derivation vs. SR derivation

12.18 On the Compton Wavelength – Quarter Period, Not Full Period

Energy-equivalently, the energy of the mass–radius coupled real object is "alternatively" defined by the Compton wavelength $\lambda_C = \frac{\pi}{2} r_0$. Thus the Compton wavelength results ≫object-naturally≪.

In "prevailing" physics, the Compton wavelength is introduced as $\lambda_C = \frac{h}{m_0 c}$, but without geometric justification and implicitly as the result of a **full period** 2π. EBT, by contrast, shows: The embodiment process from the photon to the massive elementary body corresponds to a **quarter period** $\frac{\pi}{2}$ of the underlying oscillation. Therefore the correct relation is:

$$\lambda_C = \frac{\pi}{2}\, r_0.$$

From this immediately follows the **mass-inherent radius**:

$$r_0 = \frac{2}{\pi}\,\lambda_C = \frac{2}{\pi} \cdot \frac{h}{m_0 c}.$$

The question of how "(value-)certain" the (rest) radii $r_0(m_0)$ inherent to the (rest) masses associated with the Compton wavelengths are is easy to answer: **Compton wavelengths are (also) measured quantities.** [CODATA-λ_C(proton), CODATA-λ_C(electron)]

12.19 Experimental Confirmation by Scattering Cross Sections

The following compilation shows that the **classical radius**

$$r_{\mathrm{cl}} = \frac{e^2}{4\pi\varepsilon_0 m_0 c^2}$$

derived from the mass–radius coupling appears in all established scattering formulas – a strong indication of the physical reality of the mass-inherent radius r_0. In EBT this is expressed as

$$r_{\mathrm{cl}} = \frac{\alpha}{4}\, r_0,$$

where α is the fine-structure constant. The factor $\alpha/4$ is the ratio of electrical energy to total energy.

The seven experimentally confirmed scattering formulas include:

- Thomson scattering
- Møller scattering
- Bethe–Bloch–Sternheimer equation
- Photoelectric effect
- Pair production
- Compton scattering (Klein–Nishina)
- Kramers–Heisenberg formula

In all these equations the classical radius r_{cl} or $\frac{\alpha}{4}r_0$ appears. The experimental agreement is a direct proof of the validity of the mass–radius coupling.

13 Space Energy and Space Energy Density

For macroscopic bodies the mass–radius constant equation does not hold ($M_0 R_0 \neq F_{\mathrm{EB}}$). Instead, mass-bound energy is transformed into space energy. This is the central process that makes gravitation understandable in the first place.

13.1 Space Energy

The space energy of a body of mass m_x at distance r is:

$$E_R = m_x c^2 \left[1 - \frac{m_x \gamma_G}{r c^2}\right] \tag{88}$$

Here γ_G is the gravitational constant and $r_G = m_x \gamma_G / c^2$ the gravitational radius.

Space energy is not an abstract quantity, but a real, physical form of energy that spans space. The larger the spatial extension of a body, the more mass-bound energy has been transformed into space energy.

13.2 Space Energy Density as a State Variable

The space energy density $\Phi(r)$ is defined as:

$$\Phi(r) = \frac{E_R(r)}{E_R(\infty)} = 1 - \frac{r_G}{r} \tag{89}$$

This quantity describes the energetic state of space. It is not a mathematical construct like spacetime, but a physical state variable – the normalized density of space energy.

13.3 The Effective Mass

The effective, gravitationally active mass is:

$$M_{\text{eff}} = \frac{r_G}{r} m_x \tag{90}$$

This explains the weakness of gravitation: For macroscopic bodies, $r \gg r_G$, so $M_{\text{eff}} \ll m_x$. Gravitation is not weak because the coupling constant is small – it is weak because the largest part of the mass-bound energy has been transformed into space energy.

13.4 Space Energy as "Missing" Energy – The Secret of Weak Gravitation

The apparent weakness of gravitation – especially in comparison to the electrical or strong interaction – is one of the great unsolved puzzles of standard physics. EBT solves this puzzle in a simple and elegant way.

The "missing" energy, which appears in standard physics as a deficit in the energy balance, lies in EBT **in space itself**. The space energy equation [ER] shows this immediately:

$$E_R = m_x c^2 \left[1 - \frac{m_x \gamma_G}{r c^2}\right]$$

The larger the body, the more energy has been transformed into space energy. This space energy is not "lost" – it is the energy that spans space.

For macroscopic bodies, the ratio r_G/r is extremely small. Consider the example of Earth:

- Actual radius of Earth: $r_{\text{Earth}} \approx 6.37 \times 10^6$ m
- Gravitational radius of Earth: $r_{G,\text{Earth}} = m_{\text{Earth}} \gamma_G / c^2 \approx 8.87 \times 10^{-3}$ m
- Ratio $r_G/r \approx 1.39 \times 10^{-9}$

The effective mass of Earth is thus **about one billion times smaller** than its rest mass. Only this tiny fraction of the mass is gravitationally active.

If Earth were compressed to the size of its gravitational radius (i.e., $r = r_G$), then $r_G/r = 1$ and the effective mass would equal the rest mass – gravitation would be immensely strong. **Because Earth is, however, enormously extended**, the by far largest part of its mass-bound energy has been transformed into space energy. The weakness of gravitation is therefore **not a property of an independent force, but a consequence of the large spatial extension of ordinary matter.**

13.5 The Origin of the Gravitational Constant: The Elementary Quantum G

The gravitational constant γ_G is not postulated in EBT, but derived from the properties of a special elementary body: the **Elementary Quantum G**.

The Elementary Quantum G is defined as the body **in which no transformation of mass-bound into space-bound energy has taken place** – its actual radius is equal to its gravitational radius.

From the mass–radius coupling (F1) and the condition that for the Elementary Quantum G the gravitational self-energy equals its rest energy, it follows:

$$\gamma_G \cdot m_G^2 = F_{\text{EB}} \cdot c^2 \tag{91}$$

$$m_G \cdot r_G = F_{\text{EB}} = \frac{2h}{\pi c} \tag{92}$$

With the numerical value (CODATA 2022):

$$F_{\text{EB}} = 1.4070691767 \times 10^{-42} \text{ kg·m} \tag{93}$$

the following results for the elementary quantum:

$$r_G = \sqrt{\frac{F_{\text{EB}} \cdot \gamma_G}{c^2}} = 2 \cdot \sqrt{\frac{h\gamma_G}{2\pi c^3}} = 2r_{\text{Planck}} \tag{94}$$

$$m_G = \sqrt{\frac{F_{\text{EB}} \cdot c^2}{\gamma_G}} = 2 \cdot \sqrt{\frac{hc}{2\pi\gamma_G}} = 2m_{\text{Planck}} \tag{95}$$

The gravitational constant is thus:

$$\gamma_G = \frac{r_G}{m_G} \cdot c^2 \quad \text{(embodied gravitational constant)} \tag{96}$$

The gravitational constant in EBT is **not a fundamental constant**, but a **derived quantity**. It is the ”embodied ratio” of radius to mass of the fundamental body G.

The gravitational constant γ_G used in the ”known” Newtonian law of gravitation refers to the **length-smallest, mass-richest** body G (Elementary Quantum). This fact is not obvious, since the usually formulated law of gravitation does not explicitly reveal this original connection.

13.6 The Cross-Scale Significance of the Elementary Quantum

The Elementary Quantum G has an extraordinary significance that extends from the microcosm to the universe.

Consider the ratios:

- Mass of the universe: $M_{\text{Uni}} \approx 10^{53}$ kg
- Radius of the universe: $R_{\text{Uni}} \approx 10^{26}$ m
- Mass of the elementary quantum: $m_G \approx 10^{-8}$ kg
- Radius of the elementary quantum: $r_G \approx 10^{-35}$ m

The ratios are:

$$\frac{M_{\text{Uni}}}{m_G} \approx 10^{61} \tag{97}$$

$$\frac{R_{\text{Uni}}}{r_G} \approx 10^{61} \tag{98}$$

That means: **The ratio of mass to radius is identical for the universe and for the elementary quantum.**

EBT formulates this as the cosmic constant equation:

$$\frac{r(t)}{m(t)} \cdot c^2 = \frac{r_{\text{Uni}}}{m_{\text{Uni}}} \cdot c^2 = \gamma_G = \frac{r_G}{m_G} \cdot c^2 = \frac{r_{\text{Planck}}}{m_{\text{Planck}}} \cdot c^2 \tag{99}$$

This equation states: **The ratio of radius to mass is constant on all scales – from the elementary quantum to the universe.**

This correspondence is no coincidence. It shows that EBT provides a **unified description** of all scales – from the microcosm to the macrocosm.

The gravitational self-energy of the universe equals the rest energy of the universe:

$$E_G(\text{Uni}) = M_{\text{Uni}} c^2 \tag{100}$$

The universe is in this sense the "mirror-image counterpart" to the elementary quantum – the **length-largest** body that exhibits the same radius–mass ratio as the length-smallest body.

14 The Axioms of Elementary Body Theory

EBT is based on three phenomenological axioms that follow from the geometry of the spherical surface dynamics. They are not arbitrary postulates, but the necessary formalization of the primary, sensually experienceable quantity of radial extension.

14.1 Axiom 1: Space Energy

Physical space possesses a scalar space energy density $\Phi(\mathbf{x}) \in (0, 1]$, defined as the ratio of the local space energy to the space energy at infinite distance:

$$\Phi(\mathbf{x}) = \frac{E_R(\mathbf{x})}{E_R(\infty)} \qquad \text{(A1)} \tag{101}$$

This axiom states: Space is not an empty container and not a mathematical construct. It is a physical medium with a real, measurable energy density.

14.2 Axiom 2: Time Rate

The local proper time in a weak gravitational field is, to first order, proportional to the space energy density:

$$d\tau = \Phi\, dt + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right) \qquad \text{(A2)} \tag{102}$$

The relation $d\tau = \Phi\, dt$ is thus the first-order weak-field relation. The exact metric time scale is described by the metric effect factor $A(r)$ determined in Section 15.1:

$$d\tau = \sqrt{A(r)}\, dt, \qquad A(r) = 2\Phi(r) - 1. \tag{103}$$

Thus for the temporal metric component:

$$g_{tt} = -A(r) = -(2\Phi(r) - 1). \tag{104}$$

Time dilation in EBT is not a consequence of a curved spacetime, but an immediate consequence of the space energy state.

14.3 Theorem 3: The Material Deformation of the Measuring Instrument (The Scale Argument)

The relation

$$g_{tt}\, g_{rr} = -1 \tag{105}$$

is not an independent postulate, but describes the reciprocal scale structure of the effective EBT metric. The metric effect factor itself is not determined from the perihelion shift, but in Section 15.1 by the dynamic consistency with the static main force.

Step 1: The universal scaling of local energy

Axiom 1 defines the space energy density $\Phi(x) = E_R(x)/E_R(\infty)$. If one takes this ontology in its full scope seriously – $\Phi(x)$ is a property of space at the point x that affects every energy located there, not only that of the source –, then the rest energy of any test body (and thus also of the measuring instrument) at location r scales:

$$E(r) = E_\infty\, \Phi(r). \tag{106}$$

Step 2: The scaling of the effective mass

With the equivalence $E = m_0 c^2$ derived in Section 12 (which in EBT follows purely geometrically from the spherical surface dynamics), it follows for the local effective mass of an elementary body:

$$m(r) = m_\infty\, \Phi(r). \tag{107}$$

Step 3: The deformation of the scale

If one applies the mass–radius coupling $m(r)\,r_0(r) = F_{\mathrm{EB}}$ not to the source, but to the scale itself, which measures locally at location r, then:

$$r_0(r) = \frac{F_{\mathrm{EB}}}{m(r)} = \frac{F_{\mathrm{EB}}}{m_\infty\,\Phi(r)} = \frac{r_0(\infty)}{\Phi(r)}. \tag{108}$$

A material scale, whose elementary bodies lose effective mass in the space energy field, physically expands. It becomes larger by the factor $1/\Phi(r)$.

This material scale change justifies the reciprocal structure of the effective metric description. However, it must not be equated with the exact identification $g_{rr} = \Phi^{-2}$, since the exact metric effect factor is determined by the dynamic consistency condition.

Step 4: The Theorem

With

$$g_{tt} = -A(r) \tag{109}$$

and the reciprocal scale structure, it follows

$$g_{rr} = A(r)^{-1}, \tag{110}$$

and thus:

$$\boxed{g_{tt}\,g_{rr} = \big(-A(r)\big)\,A(r)^{-1} = -1.} \tag{111}$$

Conclusion: The relation $g_{tt}\,g_{rr} = -1$ remains the invariance of the ratio of time and radial scale of the effective EBT metric. The concrete effect factor $A(r)$ is determined independently in Section 15.1 from the consistency with the static main force. Space is not curved; rather, the local physical effect conditions in the scalar space energy field are described by an effective metric representation.

15 The Effective Metric and the Refractive Index

15.1 Derivation and Physical Motivation of the Effective EBT Metric

The metric description of the static, spherically symmetric gravitational field is not introduced in EBT as an independent geometric basic assumption (like the spacetime curvature of GR). Rather, it follows as an effective representation of the local physical conditions to which material measuring instruments (clocks and scales) are exposed in the space-energy medium.

The starting point is the scalar state variable

$$\Phi(r) = 1 - \frac{r_{G,\Phi}}{r}, \qquad r_{G,\Phi} = \frac{\gamma_G M_\Phi}{c^2}, \tag{112}$$

which describes the local space energy state.

1. Determination of the metric effect factor

The interaction energy of a test body of mass m in the external field of the source M_Φ is:

$$U_{\mathrm{stat}}(r) = mc^2\big[\Phi(r) - 1\big] = -\frac{\gamma_G M_\Phi m}{r}. \tag{113}$$

From this follows the static main force:

$$F_1(r) = -\frac{dU_{\mathrm{stat}}}{dr} = -\frac{\gamma_G M_\Phi m}{r^2}, \qquad \ddot{r} = -\frac{\gamma_G M_\Phi}{r^2}. \tag{114}$$

For a reciprocal metric representation of the form

$$ds^2 = -A(r)c^2dt^2 + A(r)^{-1}dr^2 + r^2d\Omega^2 \tag{115}$$

it follows for radial motion from the radial equation:

$$\ddot{r} = -\frac{c^2}{2}A'(r). \tag{116}$$

Equating with the static main effect yields:

$$-\frac{c^2}{2}A'(r) = -\frac{\gamma_G M_\Phi}{r^2}, \tag{117}$$

thus

$$A'(r) = \frac{2\gamma_G M_\Phi}{c^2r^2} = \frac{2r_{G,\Phi}}{r^2}. \tag{118}$$

Integration and the boundary condition $A(\infty) = 1$ give:

$$\boxed{A(r) = 1 - \frac{2r_{G,\Phi}}{r} = 2\Phi(r) - 1.} \tag{119}$$

2. The temporal component

For a local system at rest relative to the source:

$$-c^2d\tau^2 = -A(r)c^2dt^2, \tag{120}$$

and thus

$$d\tau = \sqrt{A(r)}\,dt = \sqrt{2\Phi(r) - 1}\,dt, \qquad g_{tt} = -A(r). \tag{121}$$

In the weak field:

$$\sqrt{A(r)} = \Phi(r) + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right), \tag{122}$$

so that $d\tau \approx \Phi(r)\,dt$ is the first-order weak-field relation.

Time dilation in EBT is not a property of an abstract spacetime, but the oscillation slowing of an elementary body in the space energy field.

3. The radial component

From Theorem 3 follows the reciprocal metric structure:

$$g_{rr} = A(r)^{-1} = \frac{1}{2\Phi(r) - 1}. \tag{123}$$

The product

$$g_{tt}\, g_{rr} = -1 \tag{124}$$

is thus exactly preserved.

4. The transverse spherical geometry

The radius r denotes the physical spherical radius of the static field. The space energy density modifies the local time rate and the radial propagation conditions, but not the transverse spherical geometry. For the angle element, unchanged:

$$d\ell_\perp^2 = r^2 d\Omega^2 = r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{125}$$

Thus $g_{\theta\theta} = r^2$ and $g_{\varphi\varphi} = r^2 \sin^2\theta$.

The complete effective EBT metric

The combination of these structural conditions yields the effective metric:

$$\boxed{ds^2 = -A(r)c^2dt^2 + A(r)^{-1}dr^2 + r^2 d\Omega^2, \qquad A(r) = 2\Phi(r) - 1 = 1 - \frac{2r_{G,\Phi}}{r}.} \tag{126}$$

Epistemological classification: The logical chain of EBT is:

Space energy state $\to \Phi(r) \to$ static main effect $\to A(r) = 2\Phi(r) - 1 \to$ reciprocal metric.

The metric in EBT does not have the significance of a fundamental spacetime geometry. It is the compact, effective description of the local physical effect conditions in the space energy field. Phenomena such as perihelion shift, light deflection, or Shapiro delay are not used to construct the metric, but result as consequences of this structure.

15.2 The State Equation of Space Energy

The three axioms define the phenomenological foundations of the theory. To describe the dynamics of the space energy density, however, an additional assumption is required.

As an independent postulate of EBT, the following linear state equation is introduced:

$$\nabla^2\Phi = +\frac{4\pi\gamma_G}{c^2}\,\rho\,\Phi \qquad \text{(SE)} \tag{127}$$

where ρ is the mass density of matter.

Justification of the sign and the Poisson limit:

The quantity Φ is not the Newtonian potential U, but a dimensionless space energy density of the form

$$\Phi = 1 + \frac{U}{c^2}, \qquad U = c^2(\Phi - 1). \tag{128}$$

In the weak field ($\Phi \approx 1$, i.e. $|U|/c^2 \ll 1$), (SE) reduces to:

$$\nabla^2 \Phi \approx +\frac{4\pi\gamma_G}{c^2}\,\rho. \tag{129}$$

With $U = c^2(\Phi - 1)$ and $\nabla^2\Phi = \nabla^2 U/c^2$, it follows:

$$\nabla^2 U = +4\pi\gamma_G\rho, \tag{130}$$

which is the usual Poisson equation with the correct sign for the attractive gravitational potential (in the convention $U = -\gamma_G M/r < 0$). The positive sign in (SE) is thus consistent with physical attraction.

In vacuum ($\rho = 0$), (SE) reduces to the Laplace equation $\nabla^2\Phi = 0$. The most general spherically symmetric solution with $\Phi(\infty) = 1$ is $\Phi(r) = 1 - C/r$.

15.3 Solution for a Point Mass and Source Self-Consistency

Step 1: Integration of the state equation.

We integrate (SE) over a volume V that encloses the entire mass distribution:

$$\int_V \nabla^2\Phi\, d^3x = \frac{4\pi\gamma_G}{c^2}\int_V \rho(\mathbf{x})\,\Phi(\mathbf{x})\,d^3x. \tag{131}$$

Step 2: Gauss's theorem.

$$\int_V \nabla^2\Phi\, d^3x = \oint_{\partial V} \nabla\Phi \cdot d\mathbf{A}. \tag{132}$$

For $\Phi(r) = 1 - C/r$, $d\Phi/dr = C/r^2$. The surface integral over a sphere of radius R gives:

$$\oint_{\partial V} \nabla\Phi \cdot d\mathbf{A} = \frac{C}{R^2}\cdot 4\pi R^2 = 4\pi C. \tag{133}$$

Step 3: Definition of the space-energy-weighted mass.

$$M_\Phi := \int_V \rho(\mathbf{x})\,\Phi(\mathbf{x})\,d^3x. \tag{134}$$

Step 4: Determination of the integration constant.

$$4\pi C = \frac{4\pi\gamma_G}{c^2}M_\Phi \quad\Longrightarrow\quad C = \frac{\gamma_G M_\Phi}{c^2} =: r_{G,\Phi}. \tag{135}$$

Step 5: The exact exterior solution.

$$\boxed{\Phi(r) = 1 - \frac{r_{G,\Phi}}{r}} \qquad \text{for } r \geq R_{\text{source}}, \qquad r_{G,\Phi} = \frac{\gamma_G M_\Phi}{c^2}. \tag{136}$$

Note on the range of validity: The exterior solution (136) applies exclusively for $r \geq R_{\text{source}}$. Inside an extended source ($r < R_{\text{source}}$), the state equation must be solved separately with the concrete density distribution $\rho(r)$.

EBT strong-field hypothesis (postulate): EBT postulates that for real bodies $R_{\text{source}} \geq r_{G,\Phi}$ holds and that there are no stable configurations with $R_{\text{source}} < r_{G,\Phi}$. This is an independent phenomenological assumption about the maximum compression of elementary bodies. The absence of tensorial degrees of freedom follows from the scalar nature of the state equation (SE). Whether time-dependent scalar propagation modes of space energy exist is not decided by the present static state equation alone and requires a dynamic extension of SE.

15.4 Light Propagation in the Space-Energy Medium

From the effective EBT metric, for radial light propagation ($ds^2 = 0$):

$$-A(r)c^2 dt^2 + A(r)^{-1} dr^2 = 0, \qquad A(r) = 2\Phi(r) - 1. \tag{137}$$

Step 1: Radial light velocity.

$$v(r) = \frac{dr}{dt} = c\,A(r) = c\,[2\Phi(r) - 1]. \tag{138}$$

Step 2: Refractive index.

$$\boxed{n(r) = \frac{c}{v(r)} = \frac{1}{A(r)} = \frac{1}{2\Phi(r) - 1} = \frac{1}{1 - 2r_{G,\Phi}/r}.} \tag{139}$$

Step 3: First-order expansion.

$$n(r) = 1 + \frac{2r_{G,\Phi}}{r} + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right). \tag{140}$$

Step 4: Numerical calculation for the Sun.

With the solar parameters:

- $M_\odot = 1.989 \times 10^{30}\,\text{kg}$
- $\gamma_G = 6.674 \times 10^{-11}\,\text{m}^3/(\text{kg}\,\text{s}^2)$
- $c = 2.998 \times 10^8\,\text{m/s}$

- $R_\odot = 6.957 \times 10^8\,\mathrm{m}$ (solar radius)

Gravitational radius of the Sun:

$$r_{G,\Phi} = \frac{\gamma_G M_\odot}{c^2} = \frac{6.674 \times 10^{-11} \cdot 1.989 \times 10^{30}}{(2.998 \times 10^8)^2} = 1.477 \times 10^3\,\mathrm{m}. \tag{141}$$

Ratio at the solar surface:

$$\frac{r_{G,\Phi}}{R_\odot} = \frac{1.477 \times 10^3}{6.957 \times 10^8} = 2.123 \times 10^{-6}. \tag{142}$$

Refractive index at the solar surface to first order:

$$n(R_\odot) = 1 + 2 \cdot 2.123 \times 10^{-6} = 1 + 4.246 \times 10^{-6}. \tag{143}$$

Light velocity at the solar surface:

$$v(R_\odot) = \frac{c}{n(R_\odot)} = \frac{2.998 \times 10^8}{1 + 4.246 \times 10^{-6}} \approx 2.998 \times 10^8 \left(1 - 4.246 \times 10^{-6}\right). \tag{144}$$

$$v(R_\odot) \approx c - 1.273\,\mathrm{km/s}. \tag{145}$$

Light is slowed at the solar surface by about 1.273 km/s compared to the vacuum value c. This velocity reduction is the direct consequence of the effective metric effect structure and produces the refractive index responsible for light deflection and Shapiro delay.

15.5 The Shapiro Delay

Step 1: Travel time integral.

$$\Delta t = \frac{1}{c} \int_{x_P}^{x_E} n(r)\,dx, \qquad r = \sqrt{x^2 + b^2}. \tag{146}$$

Step 2: Inserting the first-order expansion.

With

$$n(r) = 1 + \frac{2r_{G,\Phi}}{r} + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right)$$

it follows:

$$\Delta t = \frac{x_E - x_P}{c} + \frac{2r_{G,\Phi}}{c} \int_{x_P}^{x_E} \frac{dx}{\sqrt{x^2 + b^2}} + \mathcal{O}(r_{G,\Phi}^2). \tag{147}$$

Step 3: Evaluation.

With $\int dx/\sqrt{x^2 + b^2} = \ln(\sqrt{x^2 + b^2} + x)$:

$$\Delta t = \frac{x_E - x_P}{c} + \frac{2r_{G,\Phi}}{c} \ln\left(\frac{r_E + x_E}{r_P + x_P}\right) + \mathcal{O}(r_{G,\Phi}^2). \tag{148}$$

Step 4: Geometric form to first order – primary radii instead of impact parameter.

The impact parameter b is in EBT phenomenology a mere auxiliary quantity: It is defined only via the imagined, uncurved reference path of the light ray and is itself not a directly measurable thing. Consistent with the EBT basic principle of tracing secondary concepts back to the primary quantity radius, the logarithmic term can be transformed completely free of b and of the idealization $x_E \approx r_E$, $x_P \approx -r_P$.

With the hyperbolic parametrization $x = b \sinh u$, $r = b \cosh u$ (which identically satisfies $r^2 = x^2 + b^2$): $x + r = be^u$, so that the argument of the logarithm depends only on the difference $u_E - u_P$.

This difference can be expressed exactly through the three heliocentric radii r_E, r_P, and the chord length

$$R := x_E - x_P \tag{149}$$

:

$$\frac{r_E + x_E}{r_P + x_P} = \frac{r_E + r_P + R}{r_E + r_P - R}. \tag{150}$$

This transformation is algebraically exact; the Shapiro travel time itself is still evaluated here to first order in $r_{G,\Phi}/r$.

Thus the general, conjunction-independent Shapiro formula to first order is:

$$\boxed{\Delta t_{\text{one-way}} = \frac{R}{c} + \frac{2\gamma_G M_\Phi}{c^3} \ln\left(\frac{r_E + r_P + R}{r_E + r_P - R}\right) + \mathcal{O}(r_{G,\Phi}^2).} \tag{151}$$

It holds for arbitrary sender-receiver geometry relative to the Sun and requires only primary radii as well as the chord R directly computable from the orbit geometry, for example via the elongation angle ψ:

$$R^2 = r_E^2 + r_P^2 - 2 r_E r_P \cos\psi. \tag{152}$$

Limit case (superior conjunction).

For $b \ll r_E, r_P$: $R \approx r_E + r_P - \delta$ with

$$\delta = \frac{b^2 (r_E + r_P)}{2 r_E r_P} \ll r_E + r_P,$$

so that

$$\frac{r_E + r_P + R}{r_E + r_P - R} \approx \frac{2(r_E + r_P)}{\delta} = \frac{4 r_E r_P}{b^2}. \tag{153}$$

This reproduces the approximation formula:

$$\boxed{\Delta t^{(\text{approximation})}_{\text{Shapiro, one-way}} = \frac{2\gamma_G M_\Phi}{c^3} \ln\left(\frac{4 r_E r_P}{b^2}\right).} \tag{154}$$

Step 5: Numerical calculation for a radar echo Earth–Venus.

With the parameters of the solar system:

- $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$
- $\gamma_G = 6.674 \times 10^{-11}\,\mathrm{m^3/(kg\,s^2)}$
- $c = 2.998 \times 10^8\,\mathrm{m/s}$
- $r_E = 1.496 \times 10^{11}\,\mathrm{m}$ (Earth's orbit, 1 AU)
- $r_P = 1.082 \times 10^{11}\,\mathrm{m}$ (Venus's orbit)
- superior conjunction, minimum approach at $b = R_\odot = 6.957 \times 10^8\,\mathrm{m}$

Gravitational radius of the Sun and prefactor:

$$r_{G,\odot} = \frac{\gamma_G M_\odot}{c^2} = 1.477 \times 10^3\,\mathrm{m}, \tag{155}$$

$$\frac{2r_{G,\odot}}{c} = 9.853 \times 10^{-6}\,\mathrm{s} = 9.853\,\mu\mathrm{s}. \tag{156}$$

With $R = r_E + r_P - \delta$ (calculated exactly from x_E, x_P, b) the geometric form gives:

$$\ln\!\left(\frac{r_E + r_P + R}{r_E + r_P - R}\right) = 11.8039, \tag{157}$$

compared to $\ln(4r_E r_P/b^2) = 11.8039$ from the approximation formula. The difference for $b \ll r_E, r_P$ is at the 8th decimal place and is thus numerically meaningless. The gain of the geometric form is structural: It replaces the auxiliary quantity b by primary, directly measurable radii and remains valid outside the conjunction approximation within the first gravitational order used here.

Delay for the one-way path:

$$\Delta t_{\text{one-way}} \approx 9.853\,\mu\mathrm{s} \cdot 11.804 \approx 116.30\,\mu\mathrm{s}. \tag{158}$$

Since Shapiro measurements are radar echoes, the observable delay is the round-trip:

$$\Delta t_{\text{Shapiro, round-trip}} = 2\Delta t_{\text{one-way}} \approx 232.60\,\mu\mathrm{s}. \tag{159}$$

Depending on the exact planetary constellation (e.g., Mercury instead of Venus, different elongation ψ) and the actually achievable minimum impact parameter, this value varies; in the literature, for various probe/conjunction configurations, values between about 230 and 250 μs are given. The value calculated here of 232.6 μs lies in this order of magnitude; an exact agreement with a specific literature value would require the exact knowledge of the respective conjunction geometry (R or ψ at the observation time) and is not claimed here.

Classification. The Shapiro delay follows in EBT from the exact refractive index

$$n(r) = \frac{1}{2\Phi(r) - 1}, \tag{160}$$

whose first-order expansion

$$n(r) = 1 + \frac{2r_{G,\Phi}}{r} + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right) \tag{161}$$

provides the travel time correction used above. Numerically, the result in this order remains unchanged compared to version 17. The Shapiro delay is thus a consistency test of the effective EBT metric in the post-Newtonian regime.

16 The Complete Logical Chain

Step	Operation	Result
0	Freyling intervention	$r(t) = r_0 \sin(ct/r_0)$, $v(t) = c\cos(ct/r_0)$
1	Evolution	$t_0 = \pi r_0/(2c)$, $\lambda = \pi r_0/2$
2	Compton identification	$m_0 r_0 = 2h/(\pi c) = F_{\mathrm{EB}}$
3	Axioms + Theorem 3	$\Phi = E_R/E_R(\infty)$, $d\tau \simeq \Phi\, dt$ (1st order), $g_{tt}g_{rr} = -1$
4	Metric	$ds^2 = -Ac^2dt^2 + A^{-1}dr^2 + r^2d\Omega^2$, $A = 2\Phi - 1$
5	State equation	$\nabla^2\Phi = +4\pi\gamma_G\rho\Phi/c^2$
6	Source self-consistency	$M_\Phi = \int \rho\Phi\, d^3x$; $\Phi = 1 - r_{G,\Phi}/r$
7	Light propagation	$v(r) = cA(r) = c[2\Phi(r) - 1]$
8	Refractive index	$n(r) = 1/A(r) = 1/[2\Phi(r) - 1]$
9	Shapiro	$\Delta t = 2\gamma_G M_\Phi/c^3 \cdot \ln(4r_E r_P/b^2)$ (1st order)

Table 7: The complete logical chain of the EBT derivation

17 The Hierarchical Structure of EBT Gravitation

The gravitational description of Elementary Body Theory distinguishes two physical levels: the formation of the gravitational source structure through the mass–space coupling on the one hand, and the local motion of a test body in this already formed source structure on the other.

This separation is mandatory in order to avoid double counting: The space-energy self-correction of the source is already contained in the definition of the effective source mass M_Φ and must not be applied a second time as an additional local force on the test body.

17.1 The Static Main Force

The interaction energy of a test body of mass m in the external field of the source M_Φ is:

$$U_{\mathrm{stat}}(r) = mc^2\big[\Phi(r) - 1\big] = -\frac{\gamma_G M_\Phi m}{r}. \tag{162}$$

The static main force follows by differentiation:

$$\boxed{F_{\mathrm{stat}}(r) = -\frac{dU_{\mathrm{stat}}}{dr} = -\frac{\gamma_G M_\Phi m}{r^2}.} \tag{163}$$

This term formally corresponds to Newton's law of gravitation. Within EBT it is the direct consequence of the mass–space coupling.

Clarification on the effective mass: The quantity $M_{\mathrm{eff}} = (r_G/r)m_x$ introduced in Chapters 3 and 10 describes the portion of mass-bound energy that still exists as bound mass after the transformation into space energy. It is a measure of the gravitational weakness of macroscopic bodies and not the source mass in the force law (163). The source mass in the external field is M_Φ.

17.2 The Orbit Equation from the EBT Metric

The EBT metric is:

$$ds^2 = -A(r)c^2dt^2 + A(r)^{-1}dr^2 + r^2d\Omega^2, \qquad A(r) = 2\Phi(r) - 1 = 1 - \frac{2r_{G,\Phi}}{r}, \tag{164}$$

where

$$r_{G,\Phi} = \frac{\gamma_G M_\Phi}{c^2}. \tag{165}$$

The decisive point: The angle term is $r^2d\Omega^2$. It is not modified. Space energy acts on the temporal component ($g_{tt} = -A$) and on the radial component ($g_{rr} = A^{-1}$), but the angle geometry remains that of the spherical surface with radius r.

Step 1: Conserved quantities from the symmetries of the metric.

Since the metric is independent of t and φ, two exact conserved quantities exist. For the motion of a test body in the equatorial plane ($\theta = \pi/2$):

$$E = A(r)c^2\dot{t}, \tag{166}$$

$$\ell = r^2\dot{\varphi}. \tag{167}$$

The specific canonical angular momentum is exactly $\ell = r^2\dot{\varphi}$, without any correction factor. This is the direct consequence of the fact that the angle term of the metric is $r^2d\varphi^2$.

Step 2: The normalization condition.

For a timelike orbit: $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -c^2$:

$$-c^2 = -Ac^2\dot{t}^2 + A^{-1}\dot{r}^2 + r^2\dot{\varphi}^2. \tag{168}$$

Inserting the conserved quantities:

$$-c^2 = -\frac{E^2}{Ac^2} + A^{-1}\dot{r}^2 + \frac{\ell^2}{r^2}. \tag{169}$$

Multiplication by A:

$$-Ac^2 = -\frac{E^2}{c^2} + \dot{r}^2 + A\frac{\ell^2}{r^2}. \tag{170}$$

Solving for $\dot{r}^2$:

$$\boxed{\dot{r}^2 = \frac{E^2}{c^2} - A(r)\left(c^2 + \frac{\ell^2}{r^2}\right).} \tag{171}$$

Step 3: The effective potential.

Equation (171) has the structure of a one-dimensional energy conservation:

$$\dot{r}^2 = \frac{E^2}{c^2} - U_{\text{eff}}(r), \tag{172}$$

with the effective potential:

$$U_{\text{eff}}(r) = A(r)\left(c^2 + \frac{\ell^2}{r^2}\right). \tag{173}$$

Step 4: Exact evaluation with $A(r) = 1 - 2r_{G,\Phi}/r$.

Since $A(r)$ is exactly linear in $r_{G,\Phi}/r$, no truncation of the metric is required:

$$U_{\text{eff}}(r) = \left(1 - \frac{2r_{G,\Phi}}{r}\right)\left(c^2 + \frac{\ell^2}{r^2}\right). \tag{174}$$

Multiplied out:

$$U_{\text{eff}}(r) = c^2 + \frac{\ell^2}{r^2} - \frac{2r_{G,\Phi}c^2}{r} - \frac{2r_{G,\Phi}\ell^2}{r^3}. \tag{175}$$

With $r_{G,\Phi} = \gamma_G M_\Phi/c^2$:

$$U_{\text{eff}}(r) = c^2 + \frac{\ell^2}{r^2} - \frac{2\gamma_G M_\Phi}{r} - \frac{2\gamma_G M_\Phi \ell^2}{c^2 r^3}. \tag{176}$$

The term $-2\gamma_G M_\Phi/r$ is the Newtonian potential (up to the conventional factor 2, which comes from the normalization). The term $-2\gamma_G M_\Phi \ell^2/(c^2 r^3)$ is the transverse contribution. It arises from the exact metric structure and not as a separate postulate.

Step 5: The Binet equation.

With the substitution

$$u = \frac{1}{r}, \qquad \dot{r} = -\ell u' \tag{177}$$

the radial energy equation becomes:

$$\ell^2 (u')^2 = \frac{E^2}{c^2} - c^2 - \ell^2 u^2 + 2r_{G,\Phi}c^2 u + 2r_{G,\Phi}\ell^2 u^3. \tag{178}$$

Differentiation with respect to φ yields:

$$2\ell^2 u' u'' = -2\ell^2 u u' + 2r_{G,\Phi}c^2 u' + 6r_{G,\Phi}\ell^2 u^2 u'. \tag{179}$$

After division by $2\ell^2 u'$:

$$\boxed{u'' + u = \frac{\gamma_G M_\Phi}{\ell^2} + \frac{3\gamma_G M_\Phi}{c^2}\, u^2.} \tag{180}$$

The factor 3 arises from the cubic coupling term $2r_{G,\Phi}\ell^2 u^3$. The term $3\gamma_G M_\Phi u^2/c^2$ describes the transverse motion coupling resulting from the EBT metric.

17.3 The Radial Equation

From the differentiation of (171) with respect to τ, with

$$A(r) = 1 - \frac{2r_{G,\Phi}}{r}, \qquad A'(r) = \frac{2r_{G,\Phi}}{r^2} \tag{181}$$

the radial equation follows:

$$\boxed{\ddot{r} - \frac{\ell^2}{r^3} = -\frac{\gamma_G M_\Phi}{r^2} - \frac{3\gamma_G M_\Phi \ell^2}{c^2 r^4}.} \tag{182}$$

This equation follows here exactly from the metric effect factor $A(r) = 2\Phi(r) - 1$; a truncation of Φ^2 is not required.

The factor 3 in the radial equation is the mathematical consequence of the coupling of the metric effect factor with the transverse motion component. It corresponds to the same cubic coupling term that generates the contribution $3\gamma_G M_\Phi u^2/c^2$ in the Binet equation.

The radial equation (182) is the basis for the perihelion shift of Mercury.

18 Applications and Epistemological Classification

18.1 Perihelion Shift of Mercury – The Geometric Derivation

The perihelion shift of planetary orbits follows in Elementary Body Theory (EBT) as a consequence of the mass–space coupling and the metric effect structure determined from it. The derivation is based on the requirement of dynamic consistency between the static main force and the effective metric of the central field.

18.1.1 Step 1: Determination of the metric effect factor $A(r)$

The starting point is the space-energy interaction of a test body of mass m in the external field of a source M_Φ. The state variable of space energy is defined as:

$$\Phi(r) = 1 - \frac{r_{G,\Phi}}{r}, \qquad r_{G,\Phi} = \frac{\gamma_G M_\Phi}{c^2}. \tag{183}$$

From this follows the potential energy of the mass–space coupling:

$$U_{\text{stat}}(r) = mc^2\big[\Phi(r) - 1\big] = -\frac{\gamma_G M_\Phi m}{r}. \tag{184}$$

The resulting static main force F_1 determines the acceleration in the radial limit case ($\ell = 0$):

$$F_1(r) = -\frac{dU_{\text{stat}}}{dr} = -\frac{\gamma_G M_\Phi m}{r^2}, \qquad \ddot{r} = -\frac{\gamma_G M_\Phi}{r^2}. \tag{185}$$

In order to map this physical effect in a metric representation

$$ds^2 = -A(r)c^2 dt^2 + A(r)^{-1} dr^2 + r^2 d\Omega^2 \tag{186}$$

the effect factor $A(r)$ must reproduce the same radial acceleration. For radial motion, from the metric radial equation after differentiation with respect to proper time:

$$\ddot{r} = -\frac{c^2}{2} A'(r). \tag{187}$$

Equating with the independently determined static EBT main effect yields:

$$-\frac{c^2}{2}A'(r) = -\frac{\gamma_G M_\Phi}{r^2}, \qquad A'(r) = \frac{2\gamma_G M_\Phi}{c^2 r^2} = \frac{2r_{G,\Phi}}{r^2}. \tag{188}$$

Integration initially gives

$$A(r) = -\frac{2r_{G,\Phi}}{r} + C. \tag{189}$$

Under the boundary condition of a flat, undisturbed space at infinity

$$A(\infty) = 1 \tag{190}$$

$C = 1$ follows. Thus the metric effect factor is:

$$\boxed{A(r) = 1 - \frac{2r_{G,\Phi}}{r} = 2\Phi(r) - 1.} \tag{191}$$

Distinction between the two quantities: $\Phi(r)$ is the space-energy state variable. The metric effect factor is by contrast

$$A(r) = 2\Phi(r) - 1. \tag{192}$$

The two quantities are therefore not identical.

18.1.2 Step 2: The exact orbit equation (Binet equation)

The equation of motion follows from the normalization of the four-velocity

$$g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = -c^2. \tag{193}$$

Using the conserved quantities energy E and specific angular momentum ℓ, the radial energy equation follows:

$$\dot{r}^2 = \frac{E^2}{c^2} - A(r)\left(c^2 + \frac{\ell^2}{r^2}\right). \tag{194}$$

With the Binet substitution

$$u = \frac{1}{r}, \qquad \dot{r} = -\ell u', \qquad u' = \frac{du}{d\varphi}, \tag{195}$$

and

$$A(u) = 1 - 2r_{G,\Phi}u \tag{196}$$

the equation transforms to:

$$\ell^2(u')^2 = \frac{E^2}{c^2} - \big(1 - 2r_{G,\Phi}u\big)\big(c^2 + \ell^2u^2\big). \tag{197}$$

Multiplied out, the radial energy balance becomes visible:

$$\ell^2(u')^2 = \frac{E^2}{c^2} - c^2 - \ell^2u^2 + 2r_{G,\Phi}c^2u + 2r_{G,\Phi}\ell^2u^3. \tag{198}$$

Differentiation with respect to the orbital angle φ yields:

$$2\ell^2 u' u'' = -2\ell^2 u u' + 2r_{G,\Phi} c^2 u' + 6r_{G,\Phi} \ell^2 u^2 u'. \tag{199}$$

After division by $2\ell^2 u'$:

$$u'' = -u + \frac{r_{G,\Phi} c^2}{\ell^2} + 3r_{G,\Phi} u^2. \tag{200}$$

With

$$r_{G,\Phi} c^2 = \gamma_G M_\Phi \tag{201}$$

we obtain, without truncation, the exact Binet equation of EBT:

$$\boxed{u'' + u = \frac{\gamma_G M_\Phi}{\ell^2} + 3r_{G,\Phi} u^2.} \tag{202}$$

18.2 The EBT Chain

The derivation path forms a self-contained logical chain:

Step	**Operation**	**Result**
1	Space-energy state	$\Phi(r) = 1 - r_{G,\Phi}/r$
2	Force consistency	$g_{tt} = -(2\Phi - 1)$
3	Scale invariance	$g_{rr} = (2\Phi - 1)^{-1}$
4	Orbit dynamics	$u'' + u = k + 3r_{G,\Phi} u^2$, with $k = \gamma_G M_\Phi/\ell^2$
5	Geometric solution	$\Delta\varphi \approx 3\pi r_{G,\Phi}(1/r_{\min} + 1/r_{\max})$

Table 8: The EBT chain of the geometric derivation of the perihelion shift.

18.3 Epistemological Independence of the Perihelion Shift

Since the metric effect factor $A(r)$ is determined independently of the perihelion shift by the consistency with the static main effect, and the orbit equation subsequently follows from this metric, the observed perihelion shift represents an **independent test** of the thus determined metric EBT structure.

The agreement with astronomical data is thus not the result of a mathematical "fitting", but **independently supports** the thus determined scalar-founded metric structure.

It should be noted: The orbit equation does not follow from the static main force alone. The static main effect determines the metric effect factor $A(r)$; for the complete orbit dynamics, additionally the reciprocal metric structure

$$g_{rr} = A(r)^{-1} \tag{203}$$

and the unchanged angle term

$$r^2 d\Omega^2 \tag{204}$$

are used. Precisely because of this, the logical order of the derivation remains transparent.

18.4 The Scale Argument and the Time Rate

The metric time scale is directly linked to the local space energy state. The proper time $d\tau$ of a stationary oscillator in the field is given by:

$$d\tau = \sqrt{A(r)}\,dt = \sqrt{2\Phi - 1}\,dt = \sqrt{1 - \frac{2r_{G,\Phi}}{r}}\,dt. \tag{205}$$

For weak gravitational fields with

$$x = \frac{r_{G,\Phi}}{r} \ll 1 \tag{206}$$

the Taylor expansion holds:

$$\sqrt{1 - 2x} = 1 - x - \frac{x^2}{2} + \mathcal{O}(x^3). \tag{207}$$

Thus to first order:

$$\sqrt{A(r)} = 1 - \frac{r_{G,\Phi}}{r} + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right) = \Phi(r) + \mathcal{O}\left(\frac{r_{G,\Phi}^2}{r^2}\right). \tag{208}$$

The relation

$$d\tau \approx \Phi(r)\,dt \tag{209}$$

is thus the correct first-order weak-field relation, while the orbit dynamics uses the complete metric form $A(r) = 2\Phi(r) - 1$.

18.5 The State Equation of Space Energy

The spatial structure of $\Phi(r)$ is determined by the linear, scalar state equation:

$$\nabla^2\Phi = \frac{4\pi\gamma_G}{c^2}\rho_\Phi. \tag{210}$$

Here ρ_Φ denotes the space-energy-weighted mass density. Integration over the volume yields the effective source mass

$$M_\Phi = \int \rho_\Phi\, d^3x, \tag{211}$$

from which the integration constant

$$r_{G,\Phi} = \frac{\gamma_G M_\Phi}{c^2} \tag{212}$$

of the exterior solution consistently follows.

18.6 The Self-Sufficient Path via Turning Points

EBT determines the integration constants E and ℓ directly from the physical measurement quantities of the orbit. At the two turning points – perihelion $r_{\min}$ and aphelion $r_{\max}$ – the radial velocity vanishes:

$$\dot{r} = 0. \tag{213}$$

Thus the two exact determining equations hold:

$$\frac{E^2}{c^2} = A_{\min}\left(c^2 + \frac{\ell^2}{r_{\min}^2}\right) = A_{\max}\left(c^2 + \frac{\ell^2}{r_{\max}^2}\right), \tag{214}$$

where

$$A_{\text{min}} = 1 - \frac{2r_{G,\Phi}}{r_{\text{min}}}, \qquad A_{\text{max}} = 1 - \frac{2r_{G,\Phi}}{r_{\text{max}}}. \tag{215}$$

From the two turning point conditions it immediately follows:

$$\boxed{\ell^2 = c^2 \, \frac{A_{\text{max}} - A_{\text{min}}}{\dfrac{A_{\text{min}}}{r_{\text{min}}^2} - \dfrac{A_{\text{max}}}{r_{\text{max}}^2}}.} \tag{216}$$

Thus the specific angular momentum is determined from the two observable turning point radii, without needing the classical conic section relation $p = a(1 - e^2)$ as input.

In leading solar system order, it follows:

$$\frac{\gamma_G M_\Phi}{\ell^2} \approx \frac{1}{2}\left(\frac{1}{r_{\text{min}}} + \frac{1}{r_{\text{max}}}\right). \tag{217}$$

Inserting this into the precession following from the Binet equation yields:

$$\boxed{\Delta\varphi \approx 3\pi r_{G,\Phi}\left(\frac{1}{r_{\text{min}}} + \frac{1}{r_{\text{max}}}\right).} \tag{218}$$

For the precession per century this yields:

$$\Delta\varphi_{\text{Jh}} \approx 3\pi \frac{\gamma_G M_\Phi}{c^2}\left(\frac{1}{r_{\text{min}}} + \frac{1}{r_{\text{max}}}\right)\frac{100\,\text{a}}{T_{\text{orbit}}}. \tag{219}$$

With the used Mercury data

$$r_{\text{min}} \approx 4.60 \times 10^{10}\,\text{m}, \qquad r_{\text{max}} \approx 6.98 \times 10^{10}\,\text{m} \tag{220}$$

in leading solar system order:

$$\boxed{\Delta\varphi_{\text{Jh}} \approx 43.00'' \text{ per century.}} \tag{221}$$

Since both the formula and the inserted orbit data represent approximation values, the result is not stated as "numerically exact", but as leading solar system order. The complete radial equation can independently be evaluated numerically without this weak-field reduction.

18.7 The Factor 3 as a Geometric Consequence of the Orbit Dynamics

The factor 3 in the EBT orbit equation is no empirically inserted coefficient and no subsequent adaptation to the perihelion shift of Mercury. It arises directly from the coupling of the metric effect factor

$$A(r) = 1 - \frac{2r_{G,\Phi}}{r} \tag{222}$$

with the transverse motion component

$$\frac{\ell^2}{r^2}. \tag{223}$$

With $u = 1/r$ these become

$$A(u) = 1 - 2r_{G,\Phi}u \tag{224}$$

and

$$\frac{\ell^2}{r^2} = \ell^2 u^2. \tag{225}$$

Their multiplication within the radial energy equation produces the cubic coupling term:

$$2r_{G,\Phi}\ell^2 u^3. \tag{226}$$

Physically, this term describes the coupling of the radial space-energy effect to the transverse orbital motion of the test body. The specific angular momentum

$$\ell = r^2\dot{\varphi} \tag{227}$$

characterizes the transverse motion within the central field.

The mathematical origin of the factor 3 can be completely traced. From

$$\ell^2(u')^2 = \frac{E^2}{c^2} - c^2 - \ell^2 u^2 + 2r_{G,\Phi}c^2 u + 2r_{G,\Phi}\ell^2 u^3 \tag{228}$$

upon differentiation with respect to φ:

$$2\ell^2 u' u'' = -2\ell^2 u u' + 2r_{G,\Phi}c^2 u' + 6r_{G,\Phi}\ell^2 u^2 u'. \tag{229}$$

The cubic coupling term explicitly yields:

$$\frac{d}{d\varphi}\left(2r_{G,\Phi}\ell^2 u^3\right) = 6r_{G,\Phi}\ell^2 u^2 u'. \tag{230}$$

After dividing the *entire* differentiated orbit equation by

$$2\ell^2 u' \tag{231}$$

for this coupling portion:

$$\frac{6r_{G,\Phi}\ell^2 u^2 u'}{2\ell^2 u'} = 3r_{G,\Phi}u^2. \tag{232}$$

Thus the exact Binet equation is:

$$\boxed{u'' + u = \frac{\gamma_G M_\Phi}{\ell^2} + 3r_{G,\Phi}u^2.} \tag{233}$$

The factor 3 is thus no additional geometric or empirical setting. Its origin is completely determined within the derivation: It arises as a mathematical consequence of the cubic radial-transverse coupling term, which in turn follows directly from the metric effect structure determined by

$$A(r) = 2\Phi(r) - 1. \tag{234}$$

An additional decomposition of the factor 3 into hypothetical longitudinal, transverse, or three-dimensional partial contributions is neither required nor mathematically justified for the derivation.

18.8 Formal Starting Point Considering the Cosmological Constant

From today's formal perspective, it is methodologically incomplete to apply the Einstein field equation for local gravitational problems directly in the form

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{235}$$

In the cosmological standard model ΛCDM, a non-zero cosmological constant Λ is used. The more general starting point is therefore

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.} \tag{236}$$

Here

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{237}$$

so that the field equation explicitly reads

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{238}$$

For the idealized exterior field of a central mass, one continues to set

$$T_{\mu\nu} = 0 \tag{239}$$

However, for $\Lambda \neq 0$ it does not follow that

$$R_{\mu\nu} = 0, \tag{240}$$

but first

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0. \tag{241}$$

By contraction with $g^{\mu\nu}$ one obtains

$$R - \frac{1}{2}(4)R + 4\Lambda = 0, \tag{242}$$

thus

$$-R + 4\Lambda = 0 \tag{243}$$

and therefore

$$\boxed{R = 4\Lambda.} \tag{244}$$

If this result is inserted back into the vacuum equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(4\Lambda) + \Lambda g_{\mu\nu} = 0, \tag{245}$$

it follows

$$R_{\mu\nu} - 2\Lambda g_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{246}$$

and therefore

$$\boxed{R_{\mu\nu} = \Lambda g_{\mu\nu}.} \tag{247}$$

The exact static and spherically symmetric vacuum solution of this equation is therefore, for $\Lambda \neq 0$, not the pure Schwarzschild metric, but the Kottler or Schwarzschild–de Sitter metric:

$$\boxed{ds^2 = -\left(1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2.} \tag{248}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\varphi^2. \tag{249}$$

Only for a local problem in which the dimensionless contribution

$$\frac{\Lambda r^2}{3} \tag{250}$$

compared to the local central mass term

$$\frac{2GM}{c^2 r} \tag{251}$$

is negligibly small can one make the additional approximation

$$\frac{\Lambda r^2}{3} \ll \frac{2GM}{c^2 r}. \tag{252}$$

Then

$$1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3} \simeq 1 - \frac{2GM}{c^2 r} \tag{253}$$

and the Kottler metric transitions into the Schwarzschild metric used for local calculations:

$$\boxed{ds^2 \simeq -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2.} \tag{254}$$

Thus, methodologically, two statements must be distinguished:

1. The Schwarzschild metric is an exact vacuum solution of the Einstein equation under the additional setting $\Lambda = 0$.

2. If, however, the form of the Einstein equation used today in the cosmological standard model with $\Lambda \neq 0$ is taken as the starting point, the Schwarzschild metric is no longer the exact starting solution for the local central field. It arises from the Schwarzschild–de Sitter solution by neglecting the local Λ contribution.

The small numerical significance of Λ in the solar system justifies this neglect for corresponding local calculations. However, it does not justify making the step invisible in an investigation of the formal derivation. For a complete presentation of the approximation chain, the transition

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}} \tag{255}$$

$$\Downarrow \qquad T_{\mu\nu} = 0 \tag{256}$$

$$\boxed{R_{\mu\nu} = \Lambda g_{\mu\nu}} \tag{257}$$

$$\Downarrow \qquad \text{Statics and spherical symmetry} \tag{258}$$

$$\boxed{\text{Schwarzschild–de Sitter / Kottler}} \tag{259}$$

$$\Downarrow \qquad \frac{\Lambda r^2}{3} \ll \frac{2GM}{c^2 r} \tag{260}$$

$$\boxed{\text{Schwarzschild metric as local approximation}} \tag{261}$$

must be explicitly stated.

The analysis of the seamless GR derivation of the perihelion shift reveals that the so-called "prediction" is actually a back-calculation. Before even a single physical number is calculated, three critical interfaces must be passed at which Newtonian physics and idealizations are imported into the tensor apparatus.

18.9 Interface 1: The Symmetry Reduction

Before the calculation begins, seven settings are made:

1. Statics ($\partial_t g_{\mu\nu} = 0$)
2. Exact spherical symmetry
3. No rotation of the source
4. No multipole structure
5. No external bodies
6. Exterior space: $T_{\mu\nu} = 0$
7. Choice of areal radius and Schwarzschild coordinates

These are not consequences of the field equations. These are **idealizations** that completely blank out the real solar system (rotating, oblate Sun; perturbations by Venus, Jupiter, Earth). The "prediction" for Mercury is calculated in a universe that has nothing to do with reality.

18.10 Interface 2: The Identification $C = 2GM/c^2$

The vacuum equation $R_{\mu\nu} = 0$ provides no mass. It provides only a free integration constant C. The identification with the solar mass occurs through:

> *"Matching to the weak Newtonian limit."*

That is the point at which **Newton is smuggled into GR**. GR does not "know" what mass is. It must import the mass from Newtonian gravitation. Without this import, GR cannot calculate a single number.

18.11 Interface 3: The Perturbation Calculation and the Relation $p = a(1 - e^2)$

The exact Binet equation

$$u'' + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2 \tag{262}$$

is not solved exactly. Instead, a perturbation calculation around the Kepler ellipse is performed, truncated after first order. To arrive at the final formula, the relation

$$p = a(1 - e^2) \tag{263}$$

is used. This relation is a **Newtonian relation**. It holds exactly only for the Kepler ellipse. In GR, the orbit is no longer a closed ellipse – the relation is only approximately valid there. Nevertheless, it is used to obtain the formula

$$\Delta\varphi \simeq \frac{6\pi GM}{c^2 a(1-e^2)} \tag{264}$$

That is the point at which Newtonian geometry is smuggled into the GR formula.

18.12 What the GR Analysis Confirms

Aspect	What the document shows	Consequence
Symmetry reduction	7 idealizations before the first calculation	The "prediction" scenario is not reality
Mass identification	$C = 2GM/c^2$ by Newton matching	GR cannot define mass from itself
Perturbation calculation	Truncation after first order	The "exact" formula is an approximation
Relation $p = a(1-e^2)$	Newtonian geometry is imported	The formula contains Newtonian structure
Numerical evaluation	$\approx 43''$/century	The result was known before the calculation

Table 9: The three interfaces of the GR back-calculation.

Conclusion: The "prediction" of the perihelion shift is a back-calculation: One takes the known result ($43''$/century, known since Le Verrier 1859) and shows that it can be reproduced from a chain of idealizations, Newton matchings, and perturbation calculations.

18.13 Contrast: The GR Back-Calculation vs. the EBT Derivation

Aspect	GR back-calculation	EBT derivation
Symmetry reduction	7 idealizations (statics, spherical symmetry, vacuum, ...)	The same idealizations
Mass identification	$C = 2GM/c^2$ by Newton matching	$M_\Phi = \int \rho\Phi\, d^3x$ (self-consistency)
Metric structure	Birkhoff theorem (tensorial)	Theorem 3 (scale argument, scalar)
Perturbation calculation	Truncation after first order	The same perturbation calculation
Relation $p = a(1-e^2)$	Newtonian relation	The same Newtonian relation
Ontology	Spacetime curvature	Scale deformation

Table 10: Contrast: GR back-calculation vs. EBT derivation.

18.14 Summary: What Is Theorem, What Is Setting?

Element	Status in EBT	Open question
Freyling intervention	Theorem (geometric)	None
Mass–radius coupling $m_0 r_0 = F_{\text{EB}}$	Theorem (geometric)	None
Space energy E_R	Theorem (from mass–radius coupling)	None
Space energy density $\Phi(r)$	Theorem (from E_R)	None
Axiom 1 (space energy)	**Axiom** (phenomenologically founded)	None
Axiom 2 (time rate $d\tau = \Phi\, dt$)	Theorem (from oscillator dynamics)	None
Theorem 3 ($g_{tt}g_{rr} = -1$)	**Theorem** (from Axiom 1 + Axiom 2 + F_1)	None
State equation $\nabla^2\Phi = \ldots$	**Postulate**	Not derivable from Freyling interventi
Perturbation calculation / $p = a(1-e^2)$	Approximation (Newtonian relation)	Identical to GR
Factor 3 in Binet equation	Theorem (from metric structure)	Bound to Theorem 3

Table 11: Status of the elements of the EBT derivation: theorem vs. setting.

18.15 Central Forces in Physics: The Binet Equation

The Binet equation is a central tool of classical mechanics for describing the motion of a body under the influence of a central force. It establishes a direct connection between the shape of the orbit and the acting force law and is therefore of fundamental importance for understanding planetary orbits, scattering problems, and relativistic corrections such as the perihelion shift of Mercury.

18.16 The Basic Idea: Orbit Instead of Time Evolution

Instead of describing the motion of a particle as a function of time t, the Binet equation parameterizes the orbit directly by the angle θ. This is particularly useful when the force depends only on the distance r (central force) and the angular momentum $h = r^2\dot{\theta}$ is a conserved quantity.

Through this reparameterization, the equation of motion becomes a differential equation for the reciprocal distance

$$u(\theta) = \frac{1}{r(\theta)}$$

as a function of the angle.

18.17 Derivation of the Binet Equation

For a central force $F(r)$, the equation of motion in the radial direction is:

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = F(r).$$

With the specific angular momentum $h = r^2\dot{\theta} = \text{const.}$ and the substitution $u = 1/r$ as well as the conversion of time derivatives to angle derivatives, after elementary transformation one obtains the **Binet equation**:

$$\boxed{F\left(u^{-1}\right) = -m\,h^2\,u^2\left(\frac{d^2u}{d\theta^2} + u\right)} \tag{1}$$

Here:

- $F(r)$: the acting central force (positive for repulsion, negative for attraction),
- m: the mass of the orbiting body,
- $h = r^2\dot{\theta}$: the specific angular momentum per mass (constant),
- $u(\theta) = 1/r(\theta)$: the reciprocal radius as a function of the angle.

18.18 Application to the Kepler Problem

For an **attractive gravitational force**, Newton's force law holds:

$$F(r) = -\frac{GMm}{r^2}.$$

Inserting into the Binet equation (1) yields:

$$-\frac{GMm}{r^2} = -mh^2u^2\left(\frac{d^2u}{d\theta^2} + u\right).$$

With $u = 1/r$ and division by $-mh^2u^2$, the linear differential equation follows:

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}.$$

The general solution is a **conic section equation**:

$$u(\theta) = \frac{GM}{h^2}\left(1 + e\cos(\theta - \theta_0)\right),$$

which corresponds to the orbit of a planet under Newtonian gravitation – i.e., an ellipse, parabola, or hyperbola.

18.19 Why the Binet Equation Is Didactically Important

The Binet equation is a key concept for several reasons:

1. **Intuitiveness:** It translates the complex interplay of force, angular momentum, and orbit shape into a manageable differential equation.

2. **Universality:** It holds for *every* central force – from the Coulomb force to the relativistic correction.

3. **Historical significance:** It was the mathematical tool with which both Newton and Einstein analyzed planetary orbits.

4. **Didactic clarity:** It cleanly separates force law (left side) and orbit geometry (right side).

18.20 Summary

The Binet equation is the fundamental differential equation for the orbit shape under central forces. It reads:

$$\boxed{F(u^{-1}) = -mh^2u^2\left(\frac{d^2u}{d\theta^2} + u\right)}$$

In the case of Newtonian gravitation, it yields the Kepler ellipses; with the relativistic correction $\propto u^2$, it explains the perihelion shift of Mercury. It is thus an indispensable tool for understanding celestial mechanics – both in classical and relativistic physics.

Literature note: The derivation and application of the Binet equation can be found in almost every textbook on theoretical mechanics, e.g., in H. Goldstein, *Classical Mechanics*, or online in Wikipedia (keyword: *Binet equation*).

18.21 The Balance

- The EBT derivation of the perihelion shift is **more transparent** than the GR derivation, because it explicitly names its settings.
- EBT and GR use **the same mathematics** (perturbation calculation, Newtonian relation $p = a(1-e^2)$).
- The difference lies in the **ontology**: GR interprets the factor 3 as spacetime curvature, EBT as transverse motion coupling in the space-energy field.
- EBT has **only one scalar field equation** and therefore cannot determine g_{tt} and g_{rr} independently. It needs Theorem 3 (scale argument) to fix g_{rr}.
- Theorem 3 is a **theorem**, not a postulate. It follows compellingly from Axiom 1 + Axiom 2 + mass–radius coupling, applied to the scale itself. The universal scaling of local energy is **part of the definition of Axiom 1**, not an additional setting.
- The state equation $\nabla^2\Phi = +4\pi\gamma_G\rho\Phi/c^2$ is an **independent postulate** of EBT. It is not derived from the Freyling intervention.
- The GR derivation is a **back-calculation** that reproduces the known result through idealizations and Newton imports. The EBT derivation is a **deduction** from phenomenologically founded axioms and geometric theorems.

18.22 Light Deflection at the Sun

Step 1: The refractive index.

From the effective EBT metric (Theorem 3, Section 14.3), for radial light propagation follows $v(r) = c\Phi(r)^2$ and thus the refractive index:

$$n(r) = \frac{1}{\Phi(r)^2} = 1 + \frac{2r_{G,\Phi}}{r} + \mathcal{O}(r_{G,\Phi}^2/r^2). \tag{265}$$

Step 2: The deflection integral.

For a light ray with impact parameter b, x along the propagation direction, $r = \sqrt{x^2+b^2}$:

$$|\Delta\theta| = \int_{-\infty}^{+\infty} \frac{1}{n}\frac{\partial n}{\partial b}\,dx. \tag{266}$$

Step 3: Gradient of the refractive index.

$$\frac{\partial n}{\partial b} = 2r_{G,\Phi}\frac{\partial}{\partial b}\left(\frac{1}{r}\right) = -\frac{2r_{G,\Phi}\,b}{(x^2+b^2)^{3/2}}. \tag{267}$$

Step 4: Evaluation.

To first order ($n \approx 1$):

$$|\Delta\theta| = 2r_{G,\Phi}\int_{-\infty}^{+\infty} \frac{b}{(x^2+b^2)^{3/2}}\,dx = 2r_{G,\Phi}\cdot\frac{2}{b}. \tag{268}$$

$$\boxed{|\Delta\theta| = \frac{4\,r_{G,\Phi}}{b} = \frac{4\,\gamma_G M_\Phi}{c^2\,b}.} \tag{269}$$

For $b = R_\odot$: $|\Delta\theta| \approx 1.75''$.

18.23 Gravitational Redshift and GPS

18.23.1 The "GR Correction" – a Static Precorrection, Not a Dynamic Calculation

The often-quoted "GR correction" of 38 microseconds per day is not a dynamic control loop, but a static precorrection based on an approximation calculation in the weak field. The satellite clocks are set to a slightly lower frequency before launch. One calculates the expected rate difference in orbit – and corrects it in advance. This is a practical engineering solution based on a simple calculation. During operation, no continuous, dynamic "GR calculation" takes place. The clocks simply run at the preset frequency.

The GR correction is a practical engineering offset that confirms GR but does not prove it – for any theory that predicts the same offset would be compatible with GPS. The agreement with measurement is an empirical success of GR, but it is not proof of the ontological correctness of spacetime curvature.

18.23.2 The Core: Four to Five Satellites for Self-Synchronization

The satellites and receivers do not run a dynamic field theory – they solve a geometric problem. And that is precisely why the satellites are needed.

The role of the atomic clocks on board: The satellites carry high-precision atomic clocks that provide the time stamp for each transmitted signal. They are the "clock generators" and produce the signals whose travel time is measured. The frequency stability of these clocks is a necessary technical prerequisite for the functioning of the system.

The pure geometry problem: GPS is primarily a geometric trilateration problem. The GPS receiver has a simple, inexpensive quartz clock that is inaccurate. Deviations of a millionth of a second already lead to errors of 300 meters. The receiver "does not know" exactly how late it is. However, it knows the exact positions of the satellites (from the transmitted ephemeris data) and the exact time stamps of their signals.

The solution – a system of equations: The receiver measures the travel time of the signals from several satellites. From the difference in arrival times, it cannot directly calculate its position and the exact time, because it has four unknowns: latitude, longitude, altitude, and the exact time (the error of its own clock). To solve these four unknowns, it needs at least four independent equations – i.e., signals from four satellites that are not in one plane.

Mathematically, the unique solution with four satellites is only guaranteed about 50 percent of the time. Only from five satellites is the position always geometrically uniquely determinable, because unfavorable constellations (satellites in a line or plane) are avoided. This shows that the accuracy does not come from a grandiose formula, but from redundant, mutually checking geometric information.

18.23.3 The Trick: The System Corrects Itself

The receiver does not know how large the error of its clock is. But when it solves the four equations, a unique solution for position and clock error emerges. Thus the system "corrects" itself in real time – but it does not calculate GR, it solves geometry.

The "rate difference" is not an absolute value, but a relative offset. It is not dynamically relevant, but statically necessary. It is calculated once and set before launch. The system corrects itself in real time, but it does not need a dynamic GR application for this.

Why? Because the GPS system is not based on an absolute point in time, but on the difference in arrival times of the signals. The start signal sets the global zero point. Each satellite counts its own time from this zero point onward – regardless of whether these clocks run "relativistically" faster or slower. GPS would also work if all clocks ran equally fast or equally slow – as long as the relative rate difference is known and compensated. The absolute frequency is irrelevant – only the relative one counts.

The role of the ground stations: The clocks are continuously monitored and corrected by ground stations, with deviations only in the range of five billionths of a second. A crucial technical detail is that the clocks on board are not physically reset, which could lead to instabilities. Instead, the satellites send the correction parameters received from the ground together with their payload data to the receivers. The GPS receiver receives the raw time of the satellite. It uses the transmitted correction data to calculate out the error of the satellite clock. Only this corrected time is used for position determination.

Multiple systems simultaneously: Moreover, there is not only one GPS system. There are also GLONASS (Russia), Galileo (EU), and BeiDou (China). Modern receivers can use the signals from several of these systems simultaneously. The system can compare the time information from three or more sources and thus determine the most accurate and secure time – a purely practical comparison of measured values that requires no GR or alternative theories.

18.23.4 The EBT Calculation of the GPS Frequency Shift – Without Fit Parameters

18.23.4.1 The central EBT equation In EBT, an atomic clock is a physical oscillator whose frequency is determined by the local space-energy environment and the motion geometry. The frequency shift splits into two contributions:

$$\frac{\Delta f}{f_0} = \frac{\gamma_G M_\oplus}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\underbrace{\quad}_{\Delta_{\mathrm{grav}}} + \left(-\frac{v^2}{2c^2}\right)\underbrace{\quad}_{\Delta_{\mathrm{mat}}} \tag{258}$$

Both terms follow from the same EBT principle: Every form of energy – gravitational or kinetic – modifies the oscillator geometry proportional to E/E_{rest}. There is no free parameter κ.

18.23.4.2 The space-energy function $\Phi(r)$ from the mass–radius coupling The derivation of the space-energy function $\Phi(r)$ follows directly from the mass–radius coupling, without any free parameter.

Given is the mass–radius coupling:

$$m_0 \cdot r_0 = F_{EB} = \frac{2h}{\pi c} \tag{259}$$

And the gravitational parameter of Earth:

$$\mu = \frac{\gamma_G M_\oplus}{c^2} \tag{260}$$

In the gravitational field of Earth, an elementary body of mass m_0 experiences a gravitational self-energy:

$$E_{\text{grav}}(r) = -\frac{\gamma_G M_\oplus \cdot m_0}{r} \tag{261}$$

This energy modifies the effective radius of the elementary body:

$$r_{\text{eff}}(r) = r_0 + \frac{E_{\text{grav}}(r)}{m_0 c^2} \cdot r_0 \tag{262}$$

With $E_{\text{grav}} = -\gamma_G M_\oplus m_0/r$, it follows:

$$r_{\text{eff}}(r) = r_0 \left(1 - \frac{\gamma_G M_\oplus}{c^2 r}\right) = r_0 \left(1 - \frac{\mu}{r}\right) \tag{263}$$

The frequency of an oscillator is inversely proportional to its effective radius:

$$f(r) = \frac{c}{r_{\text{eff}}(r)} = \frac{c}{r_0(1 - \mu/r)} \tag{264}$$

At infinity ($r \to \infty$):

$$f(\infty) = \frac{c}{r_0} \tag{265}$$

The dimensionless space-energy function $\Phi(r)$ is the ratio of the local frequency to the frequency at infinity:

$$\Phi(r) = \frac{f(r)}{f(\infty)} = \frac{1}{1 - \mu/r} \tag{266}$$

with $\mu = \gamma_G M_\oplus / c^2$.

This is the EBT-internal space-energy function – derived from the mass–radius coupling, without any free parameter.

Remark. *Why $\kappa = 1$ is not a fit parameter: In EBT, every form of energy couples to the oscillator geometry with the same strength – proportional to E/E_{rest}. There is no geometric reason why the gravitational coupling should differ from the electromagnetic one. Gravitation in EBT is the same geometric operation (overlap of radii) as any other interaction. Therefore $\kappa = 1$ is the geometrically natural value, not an adjusted parameter.*

18.23.4.3 The gravitational contribution Numerical calculation with:

- $\gamma_G M_\oplus = 3.986 \times 10^{14}$ m^3/s^2
- $c = 2.998 \times 10^8$ m/s

- $r_1 = R_\oplus = 6.371 \times 10^6$ m (Earth's surface)
- $r_2 = R_\oplus + 20\,200 \times 10^3 = 2.6571 \times 10^7$ m (GPS orbit)

$$\mu = \frac{\gamma_G M_\oplus}{c^2} = \frac{3.986 \times 10^{14}}{(2.998 \times 10^8)^2} = 4.435 \times 10^{-3} \text{ m} \tag{268}$$

$$\left.\frac{\Delta f}{f_0}\right|_{\text{grav}} = \frac{\gamma_G M_\oplus}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{269}$$

$$= 4.435 \times 10^{-3} \cdot (1.5696 \times 10^{-7} - 3.7634 \times 10^{-8}) \tag{270}$$

$$= 4.435 \times 10^{-3} \cdot 1.1933 \times 10^{-7} = 5.293 \times 10^{-10} \tag{271}$$

Temporal effect:

$$\Delta t_{\text{grav}} = 5.293 \times 10^{-10} \times 86\,400 \text{ s/day} = +45.73\ \mu\text{s/day} \tag{272}$$

18.23.4.4 The kinetic contribution: matter wave The satellite moves at $v \approx 3\,874$ m/s. In EBT, this kinetic energy embodies itself as real space – the de Broglie matter wave:

$$\lambda_{\text{deB}} = \lambda_C \cdot \sqrt{\left(\frac{c}{v}\right)^2 - 1} \tag{273}$$

This spatial extension modifies the interaction geometry of the oscillator. The kinetic energy is:

$$E_{\text{kin}} = \frac{1}{2} m_0 v^2 \tag{274}$$

The relative modification of the transition energy is:

$$\frac{\Delta E_{\text{transition}}}{E_{\text{transition}}} = -\frac{E_{\text{kin}}}{m_0 c^2} = -\frac{v^2}{2c^2} \tag{275}$$

The negative sign: The embodied kinetic energy increases the effective radius of the oscillator and thereby reduces the transition energy (larger radii → lower energies according to $m \cdot r = F_{EB}$).

Numerical calculation:

$$\left.\frac{\Delta f}{f_0}\right|_{\text{mat}} = -\frac{v^2}{2c^2} = -\frac{(3\,874)^2}{2 \cdot (2.998 \times 10^8)^2} = -\frac{1.5008 \times 10^7}{1.7976 \times 10^{17}} = -8.349 \times 10^{-11} \tag{276}$$

Temporal effect:

$$\Delta t_{\text{mat}} = -8.349 \times 10^{-11} \times 86\,400 \text{ s/day} = -7.21\ \mu\text{s/day} \tag{277}$$

18.23.4.5 The total result

$$\Delta t_{\text{total}} = \Delta t_{\text{grav}} + \Delta t_{\text{mat}} = +45.73 - 7.21 = +38.52\ \mu\text{s/day} \tag{278}$$

Contribution	EBT formula	Value [μs/day]
Gravitation (space-energy density)	$\frac{\gamma_G M_\oplus}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \cdot 86400$	+45.73
Kinetics (matter wave)	$-\frac{v^2}{2c^2} \cdot 86400$	−7.21
Net		+38.52
Observed		+38.7

Table 12: EBT GPS frequency shift – summary of contributions.

The deviation is +0.5% – within the rounding accuracy of the input parameters.

The decisive point: No free parameters. The entire calculation uses exclusively:

- γ_G and $M_\oplus$ (gravitational parameters of Earth)
- c (speed of light)
- r_1, r_2 (geometric positions)
- v (orbital velocity of the satellite)

Not a single parameter was adjusted to the known result of +38.7 μs/day. The values +45.73 and −7.21 follow compellingly from the EBT principle:

> Every form of energy modifies the oscillator geometry proportional to E/E_{rest}.

This is the EBT translation of what standard physics calls "time dilation". In EBT, no time is stretched. The oscillator oscillates differently because its physical environment is different.

18.23.5 The EBT Prediction as a Falsifiable Claim

EBT predicts:

$$f = f(m_{\text{oscillator}},\, v,\, r,\, M_{\text{environment}},\, \lambda_{\text{deB}}) \tag{279}$$

Concretely:

$$\frac{\Delta f}{f_0} = \frac{\gamma_G M_\oplus}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \frac{v^2}{2c^2} \tag{280}$$

This equation is falsifiable:

1. It predicts a specific frequency shift for every satellite orbit.
2. It predicts a specific kinetic correction for every velocity.
3. It predicts that the frequency shift scales linearly with the gravitational potential difference and quadratically with velocity.

The confirmation or refutation of these predictions by independent measurements (without SR calibration) forms the decisive empirical touchstone of EBT versus GR.

18.23.6 Methodological Reflection: Independence of the EBT Derivation

The above EBT calculation of the GPS frequency shift raises a methodological question: Does EBT actually independently arrive at the two contributions, or are structures silently adopted from GR or SR?

A precise examination of the derivation steps shows:

The gravitational term: EBT derives it exclusively from the mass–radius coupling, the gravitational self-energy, and the resulting modification of the effective radius. It uses no Schwarzschild metric, no GR field equations, no spacetime curvature, and no geodesics.

The kinetic term: EBT derives it from the de Broglie matter wave and the relative modification of the transition energy. It uses no Lorentz transformation, no SR time dilation, and no four-velocity.

The use of classical kinetic energy $E_{\text{kin}} = \frac{1}{2}m_0v^2$ is an approximation for $v \ll c$. The exact EBT expression would be $E_{\text{kin}} = m_0c^2(1/\gamma_{\text{dyn}} - 1)$ with $\gamma_{\text{dyn}} = \sqrt{1 - v^2/c^2}$, which leads in first order to $\frac{1}{2}m_0v^2$. For GPS velocities ($v \approx 3.874$ km/s), this approximation is excellent and physically justified.

The formal agreement with GR arises because both theories provide the same effective description in the weak field – but with a completely different ontological basis:

GR	EBT
Spacetime curvature	Mass–radius coupling
↓	↓
Time dilation	Space energy
↓	↓
Frequency shift	Oscillator modification
↓	↓
Frequency shift	Frequency shift

EBT is not a reformulation of GR. It is an independent theory that yields numerically equivalent results in the limit of weak fields – but without spacetime, without metric, and without free parameters.

The cleanest formulation for scientific communication is therefore:

> "EBT constructs from its own basic assumptions a quantitative frequency shift whose leading GPS contributions have the same numerical structure and order of magnitude as the exper-

imentally relevant correction. The formal agreement with GR arises in the weak field, not from an adoption of relativistic concepts."

18.23.7 Epistemological Conclusion

Standard physics says: "Time runs faster on the satellite."

EBT says: "The oscillator on the satellite oscillates faster because it is in a lower space-energy density and its kinetic energy is embodied as a matter wave."

Both statements yield the same number (+38.5 μs/day). But the ontological interpretation is fundamentally different:

Standard physics	EBT
Time is a dimension that stretches	Time is not an independent entity
The clock measures the "stretched time"	The clock IS the physical process
Time dilation is a property of spacetime	Frequency change is a property of the oscillator
κ is "built into" the metric	$\kappa = 1$ follows from the energy–geometry coupling

Table 13: Ontological comparison: standard physics vs. EBT for the GPS effect.

EBT needs no spacetime, no metric, no curved geometry. It needs only: energy embodies itself as space, and space modifies the oscillator geometry.

And GPS "proves" neither GR nor EBT. GPS works because it solves a geometric problem – the determination of a position from travel time differences. The "relativistic correction" is a fixed offset that is programmed into the clocks before launch. It is a practical engineering solution, not a dynamic calculation. The question is not whether GPS "works" – it obviously does. The question is which ontology correctly describes the frequency shift of the atomic clocks. And EBT provides a physically intuitive, ontologically economical, and causally explanatory interpretation – without the conceptual baggage of spacetime.

18.24 The Theory-Ladenness of Astrophysical "Observations": What Is Actually Measured in a Foreign Galaxy?

Before EBT is applied to galactic scales, a radical separation between the detector signal and the reconstructed galaxy dynamics must be made. The nearest large spiral galaxy is the Andromeda galaxy (M31) at about 2.537 million light-years distance.

The direct measurement signal:

We see no rotation of a galaxy as motion in the sky. The angular motion would be far too small for that. What is actually recorded are photons or electromagnetic radiation, spatially and frequency-resolved. A spectrograph initially provides intensity as a function of wavelength or frequency and position:

$$I(x, y, \lambda) \qquad \text{or in radio astronomy} \qquad I(x, y, \nu). \tag{270}$$

For the 21-cm line of neutral hydrogen, a radio telescope measures radiation intensity as a function of frequency in the vicinity of $\nu_{21} \approx 1420.406\,\text{MHz}$. At sky position A, the detector registers an intensity maximum at ν_A; at sky position B, there is a corresponding maximum at ν_B. **That is measurement.**

Where does the interpretation as rotation begin?

One identifies certain spectral structures with known atomic transitions. Subsequently, the frequency difference is interpreted as a Doppler shift:

$$\frac{\Delta\lambda}{\lambda_0} \simeq \frac{v_{\rm los}}{c}. \tag{271}$$

For a disk galaxy, a geometric-kinematic model is added:

$$v_{\rm los}(R,\theta) = v_{\rm sys} + v_{\rm rot}(R)\,\sin i\,\cos\theta. \tag{272}$$

One needs: an assumed galaxy center, a disk orientation, the inclination angle i, a systemic velocity, a geometric assignment of the measurement point to a radius R, and usually an assumption about the type of motion.

In the classic study by Rubin and Ford (1970), for example, spectra of 67 H II regions in M31 were taken. The rotation velocities were calculated under the explicitly stated **assumption of exclusively circular motions**.

That means:

Spectrum measured $\neq$ rotation velocity reconstructed.

The four-level hierarchy:

Level	**Example**	**Status**
1. Detector signal	$I(x,y,\nu)$, F_ν	directly measured
2. Identification	spectral line $\rightarrow$ atomic transition	physical assignment
3. Kinematic reconstruction	$\Delta\nu \rightarrow v_{\rm los} \rightarrow v_{\rm rot}(R)$	model-dependent
4. Dynamic interpretation	$v(R) \rightarrow M(R) \rightarrow$ dark matter	strongly theory-dependent

Table 14: Hierarchy of epistemological levels in galactic observations

Between level 3 and 4 lies an enormously important difference. That different regions of a galaxy show systematically different spectral positions is empirically very well established. That a certain velocity structure can be reconstructed from this also has strong empirical support, but is already modeled. That this reconstructed velocity structure then requires a certain invisible mass distribution is a further interpretation.

The Milky Way as a special case:

We are inside the Milky Way and cannot simply observe its rotation curve from outside. Here too, the 21-cm line is measured. One obtains intensity distributions as a function of galactic direction and frequency: (l, b, ν, I). For parts of the Milky Way, the so-called **tangent point method** is used: From maximum radial velocities and the assumed geometry of galactic rotation, a rotation curve is constructed. Thus, even for our own galaxy, we do not simply "see" $R \rightarrow v(R)$.

Flux and luminosity:

What can be measured is a radiation flux at the detector:

$$F = \frac{\text{received energy}}{\text{area} \cdot \text{time}}, \qquad \text{or frequency-resolved: } F_\nu. \tag{273}$$

This is a real local measurement quantity. But the luminosity L is not the same:

$$L = 4\pi d^2 F. \tag{274}$$

To convert measured flux into an intrinsic luminosity, one needs a distance d. Thus the interpretation chain is:

$$F_\nu \to L_\nu \to \text{stellar population} \to M/L \to M_{\text{bar}}. \tag{275}$$

The baryonic mass $M_{\text{bar}} \approx 5 \times 10^{10}\, M_\odot$ used in the literature is therefore not a directly measured quantity, but a model-dependently reconstructed quantity.

Consequence for EBT:

The methodologically cleaner question for EBT is first:

Can EBT explain the spatially dependent spectral patterns of a galaxy?

and only then:

Is their interpretation as $v_{rot}(R)$ justified independently of EBT at all?

The standard model does not "measure" here – it "fits" an invisible dark matter halo until the Newtonian expectation agrees with the frequency measurement.

18.25 The EBT Thought Model for Disk Galaxies: A Possible Thought Model

Instead of such a fitting procedure, EBT assumes a geometric mechanism. A galaxy is not a point mass, but an extended disk. The EBT thought model assumes that the collective space-energy propagation beyond a characteristic scale R_t could transition from a 3D-spherical to a 2D-cylindrical (radial-planar) mode geometry.

This transition scale is defined as the geometric mean of the local gravitational length and the global universe scale:

$$R_t \sim \sqrt{r_{G,\Phi} \cdot r_{\text{Uni}}}. \tag{276}$$

This is a defining model assumption of EBT, not a freely adjustable parameter for individual galaxies.

Status: This is a **possible thought model**, not a fixed prediction. The 3D→2D derivation is conducted as a working hypothesis. Before explaining why galaxies "rotate flatly", it must first be precisely formulated which real detector signal is supposed to be explained at all (see Section 15.4).

18.26 The Mathematical Consequence of This Transition

In the 2D regime (for $R \gg R_t$), the vacuum state equation reduces to the 2D Laplace equation:

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\Phi}{dR}\right) = 0 \quad \Longrightarrow \quad \frac{d\Phi}{dR} \propto \frac{1}{R}. \tag{277}$$

From this follows $a(R) \propto 1/R$. For a stable circular orbit ($v^2/R = |a(R)|$):

$$\frac{v^2}{R} \propto \frac{1}{R} \quad \Rightarrow \quad v(R) \approx \text{const.} \tag{278}$$

The orbital radius cancels completely. In the collective disk-dominated region, a flat rotation curve therefore results.

By continuous matching of the acceleration at the transition radius R_t ($a_{2\mathrm{D}}(R_t) = a_{3\mathrm{D}}(R_t)$), structurally follows:

$$v_{\text{flat}}^2 = \frac{\gamma_G M_\Phi}{R_t}. \tag{279}$$

With $R_t = \xi\sqrt{r_{G,\Phi}\, r_{\text{Uni}}}$ and $r_{G,\Phi} = \gamma_G M_\Phi / c^2$:

$$v_{\text{flat}}^4 = \frac{\gamma_G^2 M_\Phi^2}{\xi^2\, r_{G,\Phi}\, r_{\text{Uni}}} = \frac{\gamma_G\, c^2}{\xi^2\, r_{\text{Uni}}}\, M_\Phi. \tag{280}$$

Thus:

$$\boxed{v_{\text{flat}}^4 \propto M_\Phi.} \tag{281}$$

In the weak galactic field ($M_\Phi \simeq M_{\text{baryon}}$), the scaling structure of the baryonic Tully–Fisher relation follows:

$$v_{\text{flat}}^4 \propto M_{\text{baryon}}. \tag{282}$$

18.27 Epistemological Conclusion on Galactic Dynamics

EBT explains the structural form of flat rotation curves and the Tully–Fisher relation as a possible consequence of the dimensional change of the space-energy mode geometry (3D $\to$ 2D), without postulating dark matter.

Important: This is a conceptual framework and a working hypothesis. The exact numerical normalization (the prefactor) depends on the real, continuous solution of the 3D$\to$2D transition for specific baryonic density profiles ($\Sigma(R)$). The precise numerical normalization to galaxy populations remains an empirical task that must follow from the solution of the extended EBT field equation for real disk geometries, not from subsequent fit factors.

18.28 Summary of the Applications

$\Phi(r) = 1 - r_{G,\Phi}/r$ is the common static basic structure of all applications. The different phenomena follow from the effective EBT metric, whose structure $g_{tt} \cdot g_{rr} = -1$ is derived as Theorem 3 (Section 14.3) from the material deformation of the scale in the space-energy field. The light propagation $v(r) = c\Phi(r)^2$ follows as a direct consequence of this metric structure. The GPS frequency shift follows from the same mass–radius coupling that underlies Theorem 3.

Phenomenon	EBT equation	Result
Perihelion shift	$\Delta\theta = 6\pi\gamma_G M_\Phi/[c^2 a(1-e^2)]$	$43''$/century
Light deflection	$\lvert\Delta\theta\rvert = 4r_{G,\Phi}/b$	$1.75''$
Shapiro	$\Delta t = 2\gamma_G M_\Phi/c^3 \cdot \ln(4r_E r_P/b^2)$	247 μs
Redshift	$\nu(r) = \nu_\infty \Phi(r)$	consistent
GPS	$\Delta t = +38.52$ μs/day	$\approx +38.7$
Galaxies	3D→2D thought model	working hypothesis

Table 15: Summary of EBT applications

19 The Cosmic Expansion

The expansion of the universe is described by the sinusoidal dynamics:

$$r(t) = r_{\text{Uni}} \sin\left(\frac{ct}{r_{\text{Uni}}}\right) \tag{283}$$

The maximum expansion is reached at:

$$t_{\max} = \frac{\pi}{2c} r_{\text{Uni}} \approx 14.6 \text{ billion years} \tag{284}$$

This explains the expansion **without dark energy** and **without inflation**. The observed accelerated expansion is in EBT not a sign of an unknown form of energy, but the natural consequence of the sinusoidal evolution of the universe as a whole.

19.1 The Current State of the Universe

The ΛCDM model estimates the current age of the universe at about 13.8 billion years. With the EBT evolution equations, this yields:

$$v_{\text{exp}} \approx 0.0858\, c, \quad m_{\text{exp}} \approx 1.1787 \times 10^{53} \text{ kg}, \quad r_{\text{exp}} \approx 8.7527 \times 10^{25} \text{ m} \tag{285}$$

Remarkable is that $v_{\text{exp}} \approx c\sqrt{\alpha}$ with $\sqrt{\alpha} \approx 0.0854$. This shows a possible fundamental connection between the fine-structure constant α and the expansion velocity of the universe – a connection between micro- and macrocosm that does not exist in standard physics.

20 Cross-Scale Explanation: EBT as a Coherent Overall Picture

Elementary Body Theory describes gravitational phenomena on different spatial scales through the same fundamental mass–space and space-energy structure. What changes are not the fundamental principles, but the respectively relevant propagation geometry of space energy and the role of the resulting contributions.

EBT gravitation is not to be understood as a simple superposition of three equal-ranking forces

$$F_{\text{total}} = F_1 + F_2 + F_3$$

Rather, two physical levels must be distinguished:

$$\boxed{F_2 : \text{ Source/space-energy self-correction}}$$

and

$$\boxed{F_{\text{orbit}} = F_1 + F_3 : \text{ local orbital dynamics.}}$$

The contribution F_2 thus remains part of EBT gravitation, but is not treated as an additional force acting on a test body parallel to F_1 and F_3.

20.1 On Small Scales: the Solar System

On the scale of the solar system, the local orbital dynamics is determined in the leading contribution by

$$F_1(r) = -\frac{\gamma_G M m}{r^2}.$$

This contribution describes the local gravitational effect of the already formed mass–space structure.

The quadratic space-energy self-correction

$$F_2(r) = -\frac{\gamma_G^2 M^2 m}{c^2 r^3}$$

has, relative to the leading contribution, the order of magnitude

$$\frac{|F_2|}{|F_1|} = \frac{r_G}{r}.$$

In the weak gravitational field of the solar system:

$$\boxed{\frac{r_G}{r} \ll 1.}$$

F_2, however, is not removed from the local orbit equation because of this smallness. Its role lies rather at the level of source/space-energy self-correction and must already be taken into account during the formation of the gravitational source structure.

To be distinguished from this static self-correction is the dynamic contribution of transverse motion:

$$F_3(r) = -\frac{3\gamma_G M m h^2}{c^2 r^4}.$$

For the local orbital dynamics, therefore:

$$\boxed{F_{\text{orbit}} = F_1 + F_3.}$$

From this follows the radial equation

$$\ddot{r} - \frac{h^2}{r^3} = -\frac{\gamma_G M}{r^2} - \frac{3\gamma_G M h^2}{c^2 r^4}.$$

With the Binet transformation:

$$u'' + u = \frac{\gamma_G M}{h^2} + \frac{3\gamma_G M}{c^2} u^2$$

and thus for the perihelion shift:

$$\boxed{\Delta\theta = \frac{6\pi\gamma_G M}{c^2 a(1-e^2)}.}$$

For Mercury this results in approximately

$$43.0''$$

per century.

EBT describes on the scale of the solar system in particular:

- the Keplerian orbital dynamics from the leading $1/r^2$ contribution,
- the perihelion shift from the transverse motion coupling,
- the light deflection from space-energy light propagation,
- the Shapiro delay from the changed signal travel time,
- the GPS frequency shift from space-energy and motion modification.

The different phenomena are not explained by mutually independent gravitational mechanisms, but as different phenomenological appearances of the same mass–space coupling.

20.2 On Medium Scales: Disk Galaxies

Also on galactic scales, the quadratic space-energy contribution F_2 does not become dominant over the leading contribution with increasing distance.

For a compact source, still

$$\frac{|F_2|}{|F_1|} = \frac{r_G}{R} \ll 1.$$

The previously used explanation that flat rotation curves arise from a far-field dominance of the $1/R^3$ term is therefore not maintained.

Likewise, the mere linear superposition of the $1/s$ fields of a finite exponential matter disk is not sufficient. Such a distribution transitions in the sufficiently large far field back into the three-dimensional structure

$$a_R \propto \frac{1}{R^2}.$$

The EBT explanation of galactic rotation dynamics is therefore attributed to the changed *propagation geometry of space energy itself.*

20.2.1 The Galactic Transition Scale

For a baryonic galaxy mass M, first

$$r_G = \frac{\gamma_G M}{c^2}.$$

EBT defines as the natural transition scale between local source-bound and collective galactic space-energy propagation

$$\boxed{R_0 = \sqrt{r_G r_{\text{Uni}}}.}$$

This relation is part of the EBT thought model and not a freely adjustable parameter for an individual galaxy.

For an exponential radial disk structure

$$\Sigma(R) = \Sigma_0 e^{-R/R_0}$$

the enclosed relative fraction is

$$f(x) = 1 - e^{-x}(1+x), \qquad x = \frac{R}{R_0}.$$

The geometric half-mass or characteristic transition radius follows from

$$f(x_{1/2}) = \frac{1}{2}.$$

The solution is

$$x_{1/2} = 1.67834699\ldots$$

and thus

$$\boxed{R_{\mathrm{tr}} = 1.67834699\, R_0 = 1.67834699\sqrt{r_G r_{\mathrm{Uni}}}.}$$

The numerical factor $1.67834699\ldots$ is therefore not a numerical adjustment, but a geometric property of the exponential disk structure.

20.2.2 Flat Rotation Curves

In the local region, first

$$a_{\mathrm{local}} = -\frac{\gamma_G M}{R^2}.$$

In the collective disk-dominated space-energy regime, by contrast, an effectively radial-planar propagation structure is assumed. The relevant propagation surface grows only linearly with R there, so that

$$a_{\mathrm{col}}(R) \propto -\frac{1}{R}$$

holds.

The normalization is fixed at the characteristic transition radius:

$$a_{\mathrm{col}}(R) = -\frac{\gamma_G M}{R_{\mathrm{tr}} R}.$$

For circular motion:

$$\frac{v^2}{R} = \frac{\gamma_G M}{R_{\mathrm{tr}} R}.$$

Thus follows

$$\boxed{v_{\mathrm{flat}}^2 = \frac{\gamma_G M}{R_{\mathrm{tr}}}}$$

and therefore

$$\boxed{v_{\mathrm{flat}} = \frac{c}{\sqrt{1.67834699}} \left(\frac{r_G}{r_{\mathrm{Uni}}}\right)^{1/4}.}$$

The orbital radius cancels completely. In the collective disk-dominated region, therefore:

$$\boxed{v(R) \approx \text{const.}}$$

and thus a flat rotation curve.

For the previously used values of the Milky Way,

$$M \simeq 10^{41}\,\text{kg}, \qquad r_{\text{Uni}} \simeq 8.75 \times 10^{25}\,\text{m},$$

it follows

$$R_0 \simeq 2.61\,\text{kpc},$$

$$R_{\text{tr}} \simeq 4.38\,\text{kpc}$$

and

$$\boxed{v_{\text{flat}} \simeq 222\,\text{km/s}.}$$

Thus the order of magnitude obtained from the EBT thought model lies directly in the range of the rotation velocity reconstructed for the Milky Way.

20.2.3 The Baryonic Tully–Fisher Relation

From

$$v_{\text{flat}}^2 = \frac{\gamma_G M}{R_{\text{tr}}}$$

and

$$R_{\text{tr}} = \xi\sqrt{r_G r_{\text{Uni}}}, \qquad \xi = 1.67834699\ldots$$

it follows

$$v_{\text{flat}}^4 = \frac{\gamma_G^2 M^2}{\xi^2 r_G r_{\text{Uni}}}.$$

With

$$r_G = \frac{\gamma_G M}{c^2}$$

it follows

$$\boxed{v_{\text{flat}}^4 = \frac{\gamma_G c^2}{\xi^2 r_{\text{Uni}}} M.}$$

Thus immediately

$$\boxed{v_{\text{flat}}^4 \propto M}$$

or

$$\boxed{M \propto v_{\text{flat}}^4.}$$

The characteristic fourth power is therefore not inserted as an empirical specification, but follows from

$$r_G \propto M,$$

$$R_0 \propto \sqrt{r_G} \propto \sqrt{M}$$

and

$$v_{\text{flat}}^2 \propto \frac{M}{R_{\text{tr}}}.$$

Flat rotation curves and the baryonic Tully–Fisher relation thus appear within this EBT construction as two phenomenological consequences of the same galactic space-energy propagation structure.

20.3 On Large Scales: Galaxy Clusters and Universe

On the scale of galaxy clusters, it cannot be assumed without further investigation that the same disk-dominated geometry as in spiral galaxies applies.

Galaxy clusters have a different spatial many-body structure. EBT must therefore consider on this scale the collective space-energy embodiment of the respective overall system.

In particular, a large-scale additional gravitational effect cannot be justified by a mere far-field dominance of the F_2 term. F_2 remains a source/space-energy self-correction and does not generate a slower $1/R$ structure for a finite source in the far field.

The concrete space-energy geometry of galaxy clusters thus remains a separate application of EBT and is not extrapolated from the disk galaxy solution.

On cosmological scales, by contrast, the universe itself is considered as a overall system. Its dynamics is described within EBT by the sinusoidal evolution

$$r(t) = r_{\mathrm{Uni}} \sin\left(\frac{ct}{r_{\mathrm{Uni}}}\right).$$

The associated radial velocity is

$$v_{\mathrm{exp}}(t) = c \cos\left(\frac{ct}{r_{\mathrm{Uni}}}\right).$$

Cosmic evolution is thus interpreted as a cyclical transformation between mass-bound energy and space energy.

Dark energy and inflation are not introduced within this thought model as additional fundamental entities.

20.4 The Unity of the Scales

The cross-scale structure of EBT can thus be summarized in the following form:

- **Fundamental level:**
$$m_0 r_0 = F_{\mathrm{EB}} = \frac{2h}{\pi c}, \qquad E(r) = \frac{F_{\mathrm{EB}} c^2}{r}.$$

- **Solar system:** The local orbital dynamics follows from
$$F_{\mathrm{orbit}} = F_1 + F_3,$$
while F_2 is classified as source/space-energy self-correction.

- **Disk galaxies:** The large-scale dynamics does not follow from a dominance of F_2, but from a transition of space-energy propagation from the local $1/R^2$ structure to a collective disk-dominated $1/R$ structure. From this follow both flat rotation curves and
$$M \propto v_{\mathrm{flat}}^4.$$

- **Galaxy clusters:** The concrete collective space-energy geometry must be determined separately and must not be automatically adopted from the disk galaxy solution.

- **Universe:** The global dynamics is described by the sinusoidal evolution of the overall system.

Thus the fundamental EBT structure does not change with the spatial scale. What changes is the concrete geometric organization of space energy.

The cross-scale EBT chain can therefore be written schematically as

$$\boxed{\text{Mass–space coupling} \longrightarrow \text{Space energy} \longrightarrow \text{scale-dependent propagation geometry} \longrightarrow \text{observable dynamics.}}$$

EBT thus does not need a new fundamental gravitational law for every spatial scale. The same mass–space structure leads under different geometric conditions to different phenomenological manifestations.

21 Vacuum Energy and the 10^{120} Discrepancy

21.1 The Theoretical Vacuum Energy Density of Quantum Field Theory

Quantum field theory calculates the vacuum energy density using the Planck mass, the speed of light, and Planck's constant of action:

$$\rho_{Va} = \frac{m_{Pl} \cdot c^3}{h^3} \cdot c^2 \quad [\mathrm{J/m^3}] \tag{QFTVE}$$

This value is estimated to be about 10^{120} times the observed vacuum energy density of $\sim 10^{-9}$ J/m^3, which represents the worst theoretical prediction in the history of physics.

21.2 Transformation into the Picture of the Elementary Quantum

Using the relations $m_G = 2m_{Pl}$ and $r_G = 2r_{Pl}$ with $m_G r_G = \frac{2h}{\pi c}$, the QFT equation can be transformed into the picture of the elementary body:

$$\left(\frac{c}{h}\right)^3 = \frac{8}{\pi^3} \cdot \frac{1}{(m_G r_G)^3} \tag{108}$$

$$\rho_{Va} = \frac{m_G^4 \cdot c^3}{16 \cdot h^3} = \frac{m_G}{2 \cdot \pi^3 \cdot r_G^3} \tag{QFTVE2}$$

With the sphere volume $V_{G,\text{sphere}} = \frac{4}{3}\pi r_G^3$:

$$\rho_{Va} = \frac{2}{3 \cdot \pi^2} \cdot \frac{m_G}{V_{G,\text{sphere}}} \tag{QFTVE3}$$

21.3 The Elementary-Body-Based Energy Density of the Universe

The mass density of the universe according to the EBT estimate is:

$$\rho_{UniEk} = \frac{m(t)}{V(t)_{\text{universe}}} \tag{109}$$

With the relations:

$$r_0 = \frac{2 \cdot c \cdot t}{\pi} \tag{rt}$$

$$m(t) = \frac{2 \cdot c^3 \cdot t}{\pi \cdot \gamma_G} \tag{MUNI}$$

$$V(t)_{\text{universe}} = \frac{4}{3}\pi r_0^3 \tag{110}$$

From this follows:

$$\rho_{Va} = \frac{4c^5}{3 \cdot \pi^3 \cdot h \cdot \gamma_G} \cdot t^2 \tag{QFTVE/EKEV}$$

For $t \approx 13.798$ billion years $= 4.35133728 \cdot 10^{17}$ s, the ratio becomes:

$$\frac{\rho_{Va}}{\rho_{UniEk}} = \frac{3 \cdot \pi^2}{2} \cdot \frac{r_{\text{Uni}} m_{\text{Uni}}}{F_{EB}} \approx 6.600812 \cdot 10^{120} \tag{UNIVE}$$

This ratio corresponds exactly to the famous 10^{120} discrepancy between QFT predictions and observed vacuum energy density.

21.4 Interpretation in EBT

Obviously, quantum field theorists, without realizing it, have extrapolated the mass–radius coupling and the resulting mass–radius constancy [F1] of microscopic bodies to cosmic proportions. The enormous discrepancy between quantum field theory and experimental reality arises because, within the framework of prevailing physics, it is not understood that an extension of elementary structures leads to an equivalent mass reduction. This can be calculated both qualitatively and quantitatively exactly using the effective mass via the gravitational energy.

The present considerations are thus an impressive indication of the mass–radius coupling propagated by EBT and its connection to macroscopic many-particle structures.

22 Comparison: GR vs. EBT

Criterion	**GR**	**EBT**
Geometric basis	Spacetime (4D, abstract)	Spherical surface (3D, concrete)
What is geometrized?	Gravitation itself	The elementary body
Status of geometry	Postulated	Derived (Freyling intervention)
Number of contributions	1 (curvature)	Hierarchical: F_2 = source self-correction; $F_{\text{orbit}} = F_1 + F_3$
Free parameters	Yes (G, Λ, etc.)	No ($m_0 r_0 = 2h/(\pi c)$)
Causal mechanism	No (only geometry)	Yes (transformation of mass into space energy)
Dark matter	Postulated	Not needed (galactic space-energy propagation geometry)
Dark energy	Postulated	Not needed (sinusoidal expansion)
Measurable basic quantity	No (spacetime)	Yes (radius r)

Table 16: Comparison of GR and EBT

23 The Causal Explanation of Gravitation

EBT offers a **causal explanation** of gravitation:

1. **Primary quantity:** Every elementary body has a mass-inherent radius r_0.
2. **Transformation:** Mass-bound energy is transformed into space energy (E_R).
3. **Interaction:** The overlap of the radii (Compton + de Broglie) is gravitation.

4. **Effective mass:** Only a fraction of the mass is gravitationally active: $M_{\text{eff}} = (r_G/r)m_x$.

GR, by contrast, postulates spacetime curvature without being able to show what is physically curved. The question ≫*Why is spacetime curved?*≪ is meaningless because spacetime is a purely mathematical construct. EBT does not even ask this question – because there is no spacetime in EBT. Instead, there is space energy that arises from the transformation of mass-bound energy.

24 Conclusion: EBT as a Coherent Overall Picture

Elementary Body Theory (EBT) offers a complete, parameter-free description of gravitation that does without spacetime curvature. EBT gravitation is not to be understood as a simple superposition of three equal-ranking forces. Rather, two physical levels must be distinguished:

$$\boxed{F_2 : \text{ Source/space-energy self-correction}} \tag{286}$$

and

$$\boxed{F_{\text{orbit}} = F_1 + F_3 : \text{ local orbital dynamics.}} \tag{287}$$

The local orbital dynamics follows from the static gravitational effect F_1 and the transverse motion coupling F_3 resulting from the EBT metric. The space-energy self-correction F_2 is already contained in the source parameter M_Φ and is not applied as an additional test-body force.

$$F_1(r) = -\frac{\gamma_G M_\Phi m}{r^2}, \qquad F_3(r) = -\frac{3\gamma_G M_\Phi m h^2}{c^2 r^4}. \tag{288}$$

This structure uniformly explains all macroscopic gravitational effects:

- **Planetary orbits:** Static gravitational main effect ($1/r^2$).
- **Perihelion shift:** Transverse motion coupling from the EBT metric ($1/r^4$).
- **Flat rotation curves of galaxies:** Galactic space-energy propagation geometry – **without dark matter**.
- **Light deflection and Shapiro delay:** Refractive index $n(r) = 1/\Phi(r)^2$.
- **Cosmic expansion:** Sinusoidal dynamics – **without dark energy**.

EBT is thus not only an alternative to GR, but a **phenomenologically founded, causal theory of gravitation** that describes the same phenomena – but without spacetime curvature, without covariance violations, and without free parameters.

24.1 EBT as a Coherent Overall Picture

With this hierarchical structure, the understanding of macroscopic structures becomes consistent:

- **On small scales (solar system):** The local orbital dynamics is determined by the static gravitational main effect and the transverse motion coupling.

- **On medium scales (galaxies):** Galactic dynamics is described by the space-energy propagation geometry and replaces the postulated dark matter.
- **On large scales (galaxy clusters, universe):** Space energy describes the binding of structures and the expansion itself – without dark energy.

EBT thus provides a **unified, geometric-physical description** of all gravitational phenomena that does without the metaphysical constructs of standard physics and understands space as an *active, physical medium.* Space in EBT is not an empty container and not a mathematical construct – it is the product of a real transformation of mass-bound energy into space energy.

25 The Standard Models of Physics: A Critical Inventory of the Ontological and Methodological Deficits of SM and ΛCDM

The following is a comprehensive, epistemologically founded critique of the standard models of physics – the Standard Model of particle physics (SM) and the cosmological standard model (ΛCDM). The analysis shows that both models rest on a multitude of free parameters, unobservable postulates, and methodologically circular self-confirmations. The work documents the methodological irrationality of the standard models.

25.1 Introduction

The present work undertakes a systematic, epistemologically founded critique of the standard models of physics – the Standard Model of particle physics (SM) and the cosmological standard model (ΛCDM). The analysis shows that both models rest on a multitude of free parameters, unobservable postulates, and methodologically circular self-confirmations. The critique extends from the mass problem of the Higgs mechanism through the unfounded substructuring by quarks and gluons to cosmological inflation and dark energy.

25.2 25 Free Parameters of the Standard Model of Particle Physics (SM)

The Standard Model of particle physics (SM) operates with a multitude of free parameters that cannot be derived from the theory itself, but must be determined experimentally:

- **3 coupling constants:**
  - of the strong interaction (α_s) – 8 gluons, color charge
  - of the electromagnetic interaction (α) – photon, electric charge
  - of the weak interaction (α_W) – W^+, W^-, Z^0
- **6 quark masses**
- **3 masses of the charged leptons** (electron, muon, tau)
- **4 angles for describing quark decays**
- **1 angle for describing CP violation in the strong interaction**
- **Mass of the Higgs boson**

- **3 masses and 4 mixing angles of massive neutrinos**

The Particle Data Group (PDG), according to a counting convention that distinguishes between particles and antiparticles as well as the many color states of quarks and gluons, counts a total of 61 elementary particles. If neutrinos are their own antiparticles, the total number of elementary particles would be 58 according to the same counting conventions. (Source)

Operating with more than 20 free parameters makes "everything" possible – this should be understandable even for people of average intelligence. Strictly speaking, the Standard Model of particle physics (SM), with currently 25 free parameters, continual reparameterization, repeated substructuring, the confinement thesis, and further ad hoc assumptions, is a **philosophical and not a physical thought model**. In other words: The far fewer, more precisely sparse, predictively capable quantum chromodynamics (QCD) as a recognized theoretical concept, viewed from the philosophy of nature, is not worth mentioning in relation to QED, since the number of free parameters and the number of postulated theoretical objects added over the years due to inconsistencies with measurement results exceed any justifiable framework.

The electroweak theory and the concept of gauge bosons as "interaction mediators" as well as "energy-conservation-violating virtuals" and not (directly) observable "energy thieves" – i.e., neutrinos – are mathematics-generated emotions of overwhelmed "scientists" that create fantasies but have long ceased to create knowledge. Real-physically and epistemologically, the SM is a **methodological zero**.

The discrepancies between experimental data and non-fitting theories were and are subsequently made to fit. SM-typical "measures" were and are the introduction of further theoretical objects including new quantum numbers.

An overview of the Standard Model of particle physics can be found "under" Review of Particle Physics by the Particle Data Group. There one finds, besides experimental methods and theory descriptions of current particle physics, fundamental information on quarks, neutrinos, form factors, ... as well as information on "even more" hypothetical particles, such as leptoquarks or axions. Astrophysical "concepts", cosmological parameters, experimental tests of gravitation theories, etc. Furthermore, (apparatus) aspects of accelerator physics, Monte Carlo simulations, and much more are presented.

The Particle Data Group (PDG) is an international collaboration that provides a comprehensive summary of particle physics and related areas of cosmology. The PDG collaboration consists of 227 authors from 159 institutions in 24 countries (as of 2017), see online. It is led by a coordination team located mainly at Lawrence Berkeley National Laboratory (LBNL), which has served as headquarters since the founding of the PDG.

The foundations of quantum chromodynamics (QCD) can be found here.

25.3 The Mass Dilemma of the Standard Model and the Cosmological Admission of Bankruptcy

Established particle physics postulates that the Higgs mechanism gives elementary particles their mass. A closer analysis, however, reveals a fundamental methodological weakness: The masses of entities such as electrons, muons, or postulated quarks are **not calculated from first principles** within the model, but must be experimentally determined as **free parameters** and manually inserted into the theory. The Higgs mechanism functions here merely as a formally consistent framework for embedding already known values, not as their physical derivation.

This explanatory deficit becomes particularly clear when considering baryonic matter (protons and neutrons). Only about **1 %** of the nucleon mass can be traced back to the Higgs coupling. The remaining **99 %** are attributed to postulated dynamic effects of the **strong interaction**, for which the SM possesses **no closed analytical solution**. Thus the origin of the mass for the majority of the visible world remains formally unfounded within the SM.

25.3.1 The "Admission of Bankruptcy" of the Cosmological Standard Model (ΛCDM)

The ΛCDM model describes a universe that consists of about **95 %** unobserved constructs: **68 %** dark energy and **27 %** dark matter. The remaining **5 %** fall to the known baryonic matter.

If one places the deficits of the SM in this context, a drastic consequence results: Of these **5 %** visible matter, particle physics can again formally parameterize only the Higgs portion (about **1 %**).

It follows arithmetically that the cosmological standard model, on the basis of its own postulates, can analytically-formally capture merely 0.05 % of the universe.

The remaining **99.95 %** of the claimed cosmic reality evade analytical penetration and are managed in the form of placeholder entities or numerical approximations.

25.3.2 Lattice QCD and Perturbation Theory as Methodological Immunization

The claim that the strong interaction is calculable by **lattice quantum chromodynamics (lattice QCD)** requires a scientific-theoretical correction. Lattice QCD is **not** a deductive analytical theory, but a **numerical approximation method**.

The FLAG report [1] states unambiguously that quark masses are free parameters of the theory that cannot be obtained from purely theoretical considerations. Since quarks cannot be measured in isolation due to confinement, their determination is necessarily coupled to comparison with experimental observables.

The process is defined in the FLAG review [1] as follows:

- Observables are calculated for a variety of **input values** of the quark masses.
- These values are **adjusted** until they reproduce the experimental results [1].
- Another observable (such as the pion decay constant or baryon masses) is necessarily needed to **fix the overall scale** [1].

Thus the model provides no independent derivation of the masses, but effectively accepts them as input parameters to force agreement with empirical data.

25.3.3 Tuning Instead of Prediction

The source documents exactly the procedure that EBT calls "result-oriented simulation". The quark masses enter the calculation as so-called **"bare parameters"**. These depend on the chosen lattice regularization and must be renormalized afterwards.

Lattice QCD explicitly uses the term **tuning** for this: *"light-quark masses are typically obtained by [...] tuning them to reproduce NGB masses"* [1]. From the perspective of Elementary Body Theory, this process represents **"circular-reasoning belief work"**. Instead of calculating physical reality from first principles, the mathematical model is iteratively modified until it reflects the already known reality. The "excellent agreement" is thus not an analytical success, but the result of a gigantic numerical adjustment on computer clusters.

25.3.4 Methodological Complexity and Inconsistencies

The FLAG review [1] also illustrates the enormous complexity necessary to obtain consistent values at all:

- **Phenomenological corrections:** Corrections for electromagnetic effects (QED) and isospin breaking must be included [1, 4]. These often rely on phenomenological models rather than QCD itself.

- **Incompatible results:** Different research collaborations such as MILC, BMW, or RBC/UKQCD arrive at partially incompatible results despite similar basic methods, e.g., in the determination of the electromagnetic self-energy coefficient ϵ [5, 6, 7, 8, 9, 10, 11].

- **Systematic uncertainties:** Error sources, such as the neglect of the charm quark in the "sea", are often only conservatively estimated, since a precise quantification is hardly possible a priori.

25.3.5 The Anatomy of the Circular Reasoning ("Tuning")

The process of parameter determination in lattice QCD, as documented in the internationally authoritative FLAG review [1], follows a logical chain that technically substantiates the accusation of circularity:

- **Input:** One begins with free parameters for the quark masses, since these are not theoretically derivable. The FLAG review [1] states:

  > "Quark masses cannot be measured directly in experiment because quarks cannot be isolated, as they are confined inside hadrons. On the other hand, quark masses are free parameters of the theory and, as such, cannot be obtained on the basis of purely theoretical considerations." [1]

- **Process:** These parameters are numerically varied until the result of the simulation agrees with the measured values. The source [1] describes this process as follows:

  > "One computes as many experimentally measurable quantities as there are quark masses. [...] They are usually computed for a variety of input values of the quark masses which are then adjusted to reproduce experiment." [1]

- **Scale fixing:** To physically calibrate the entire lattice, another measured value is used as an anchor:

  > "Another observable, such as the pion decay constant or the mass of a member of the baryon octet, must be used to fix the overall scale." [1]

- **Logical consequence:** The result of the calculation is thus not an independent proof of the theory, but a consequence of the previous "tuning". The review [1] confirms this explicitly for the light quarks:

  > "Thus, in lattice calculations, light-quark masses are typically obtained by [...] tuning them to reproduce NGB masses [Nambu–Goldstone boson masses, such as pions or kaons]." [1]

The necessity of this scale fixing is documented in the specialist literature by various collaborations that use different methods for determining the lattice spacing [3, 4, 5, 6, 7].

25.3.6 Parameterization vs. Deduction

From the perspective of a logic model, the decisive difference is whether a theory operates constitutively (deriving from principles) or descriptively (subsequently describing):

Established physics (descriptive): It uses highly complex numerical methods to map reality, but admits that fundamental quantities such as the division of masses rest on pure conventions:

> "The decomposition of the sum $L_{\mathrm{QCD}} + L_{\mathrm{QED}}$ into two parts is not unique and specifying the QCD part requires a convention. [...] Such a convention allows us to distinguish the physical mass M_P [...] from the mass $\hat{M}_P$ within QCD." [1]

The corrections for electromagnetic effects are treated in detail in the literature [14, 15, 16], which further underlines the methodological complexity. The pion–pion scattering lengths provide additional experimental input here [13]. The relation between quark masses and meson masses is described by the Gell-Mann–Oakes–Renner relation [12], which, however, itself rests on phenomenological assumptions.

In current strategy papers, this purely instrumental character of the simulations is further enshrined as a goal for the coming decades:

> "We provide predictions for an increasing set of Standard Model observables and parameters [...] These facilitate the full exploitation of experimental data at both the intensity and energy frontiers. In this document we argue that for sustained progress the community requires continued access to computing as well as storage resources at large Tier-0/1 high-performance-computing centres." [2]

The Standard Model thus does not "explain" masses causally, but uses massive computing power to subsequently fit experimental data into a parameterized mathematical corset.

Conclusion: The circular reasoning consists in the fact that lattice QCD requires experimental data to define its own parameters, and subsequently declares the agreement with precisely these data as a success of the theory. The calculations (e.g., via Monte Carlo integrations on computer clusters) are iteratively modified until they reproduce the known experimental measured values. This process of subsequent adaptation to reality ("circular-reasoning belief work") represents **no independent prediction**, but an extremely elaborate **calibration to empiricism**.

In the professional world, this "excellent agreement" is often sold as an analytical success, although methodologically it is a "gigantic numerical adjustment".

In contrast to this elaborate numerical procedure, **Elementary Body Theory (EBT)** shows that a reduction to the primary quantity of radial extension r suffices to analytically derive masses and radii (such as the proton radius or the Rydberg energy) with **zero free parameters** exactly from the fundamental mass–radius constant equation ($m_0 r_0 = 2h/\pi c$).

25.3.7 Conclusion: Postulate Systems Instead of Epistemological Theories

The analysis makes clear that the Standard Model of particle physics and the cosmological standard model are **not physical theories** in the sense of a causal reduction to primary quantities. They present themselves instead as:

- **Complex postulate systems** that, through a growing number of free parameters (currently **25** in the SM, **6** in the most minimal form for the cosmological standard model), produce an “interpretive arbitrariness”.

- **Theory-laden constructs** that mask physical explanatory need through the introduction of ever new, empirically inaccessible auxiliary assumptions (dark matter, inflation, virtual particles).

- **Mathematical artifacts** that claim internal coherence but can formally-analytically justify only a vanishingly small fraction of the postulated universe.

The Consequence

This is not only an extreme methodological weakness – it is **an epistemological declaration of bankruptcy**. No other physical theory in the history of science has ever “explained” such a small proportion of its postulated domain.

25.4 What Do Pioneers of Particle Physics Think About the Standard Model?

A statement by the 1976 Nobel laureate in physics, Burton Richter, from 2006 contains a clear answer:

> “To me, something that is considered today’s most advanced theory of particle physics is not really science. When I recently found myself on a panel with three respected theorists, I could not resist discussing what I see as the main problem in the philosophy behind the theory, which seems to have drifted into a kind of metaphysical wonderland. Simply put, many of the most current theories seem to be theological speculation and the development of models without testable consequences...”

Source: Physics Today October 2006: Theory in particle physics: Theological speculation versus practical knowledge

Burton Richter (1931–2018) received the Nobel Prize in Physics in 1976 together with Samuel C. C. Ting.

The mathematical concepts of the SM stem at their core from abstractions freed from real physics. Result: Contrary to the expedient demand of epistemological considerations, the “results” of theoretical basic physics regarding the supposed “elementary particles” are more emotionally than scientifically founded answers to the ideological question of which theoretical concept one agrees on “today and soon”. [That is probably also the origin of the Americanized term diminutives (axion, WIMP, beauty quark, charm quark, ...) as an implicit cry for help: *redeem us from our nonsense, even if we claim otherwise.*] Nature has to submit to the theoretical implications. Today it is, for example, still postulated but not detectable elementary quarks; tomorrow possibly “preon-substructured”.

Also worth reading is the Spiegel article “Sperrt das Desy zu!” from 1999, in which Hans Graßmann describes the “methodological emptiness” of DESY: “Don’t believe the Desy managers: The dull columns of numbers that Desy produces are not physics...”

If one wants to learn “something” about the practiced horrendous nonsense in theoretical and experimental high-energy physics within the framework of the Standard Model of elementary particle physics (SM), Sabine Hossenfelder’s contribution, published on 26.09.2022 in The Guardian, is recommended. Furthermore, she published a blog post on 28.09.2022 about it.

Below, as examples, some statements in the Guardian to understand why Ms. Hossenfelder is “sharply attacked" by the apologists and epigones of the SM:

> "In private, many physicists admit that they do not believe in the existence of the particles they are paid to search for..."

> "Imagine you attend a zoology conference. The first speaker talks about her 3D model of a 12-legged purple spider that lives in the Arctic. She admits there is no evidence for its existence, but it is a testable hypothesis, and she advocates sending a mission to search for spiders in the Arctic. The second speaker has a model for a flying earthworm, but it only flies in caves. There is no evidence for this either, but he advocates searching the world's caves. The third speaker has a model for octopuses on Mars. It is testable, he emphasizes. Hats off to the zoologists, I have never heard of such a conference. But at almost every particle physics conference there are sessions like this, except that they involve more mathematics."

> "It has become common among physicists to invent new particles for which there is no evidence, publish papers about them, write further papers about the properties of these particles, and demand that the hypothesis be tested experimentally. Many of these tests have already been carried out, and more are being commissioned. This is a waste of time and money."

> "As a former particle physicist, it makes me sad to see that this field has become a factory for useless academic studies."

25.5 Confinement and Asymptotic Freedom – Epistemology Adieu

The "problems" with the theoretical deeds of Frank (Anthony) Wilczek, born 1951, possess unique characteristics of a special kind. He preferred to explain the incomprehension of particle physics metaphysically with the help of a new theoretical object, which he named *axion* after an American laundry detergent (Americans bleached their underwear with it in the 1970s). Wilczek received the Nobel Prize in Physics in 2004 together with David Gross and H. David Politzer for the introduction of asymptotic freedom into the theory of the strong interaction.

In the "scientific paper" Asymptotic Freedom – From Paradox to Paradigm, Wilczek writes, among other things, in the introduction:

> "In theoretical physics, paradoxes are good. That's paradoxical, since a paradox appears to be a contradiction, and contradictions imply serious error. But Nature cannot realize contradictions. When our physical theories lead to paradox we must find a way out. Paradoxes focus our attention, and we think harder."

Then under 1.1 he writes: "Quarks are Born Free, but Everywhere They are in Chains."

So much for the missing linguistic possibilities and Wilczek's wish to compulsively anthropomorphize factual contexts. Wilczek's infantile manner is reminiscent of a would-be comedian striving for human closeness. That is sad enough for entertainment reasons. But the formative works resulting from his contributions to the theory of matter formation are the actual problem. Asymptotic freedom and the confinement thesis carry the edifice of quantum chromodynamics. Simply put: From an epistemological viewpoint, these postulates do not solve the phenomenological problems, but simply elevate the actual failure of the theory to a theory.

As an aside, it can be noted that not even within the arbitrariness-affine SM is there clarity. Regarding confinement, the English Wikipedia says: “There is not yet an analytic proof of color confinement in any non-abelian gauge theory.” The German version reads: “A complete theoretical description of this experimental finding is still pending.”

25.6 Beginning and End of the SM – Fatal Theory Errors Proven by the SM Itself

25.6.1 Quarks Are Not Fermions – The Non-Existent Spin of Quarks and Gluons

I would like to and “must” not go into technical detail here, because it is not necessary, since it can be shown, so to speak, easily that from overarching rational-logical reasons the thesis that quarks are fermions (and especially within the structure and implications of the SM) can be clearly refuted, since corresponding Standard Model-induced experiments exist that state this.

And here we have a concrete situation that is extremely difficult to “cope with”. Psychologically at first almost insoluble, because no one can imagine that a supporting hypothesis of a thought model established over decades is demonstrably false and is not corrected as such. Tens of thousands of well-trained scientists over several generations live and work with a serious false statement? In particular laypeople, but also physicists who are not familiar with the formalism of the SM, simply cannot imagine that this is possible. One assumes that the critical observer who draws attention to the verifiable connections has overlooked something, emotionally favored by the circumstance that one does not oneself possess the necessary mathematical-formal expertise. These are extremely difficult boundary conditions from the perspective of reality-seeking enlightenment. Nevertheless, I am of the opinion that the state of affairs can very well be explained and understood. I will try:

The first assumption was, due to the theoretical specifications in the mid-1960s, that in the picture of the SM the postulated proton spin is composed to 100 % of the spin components of the quarks. This assumption was not confirmed in the EMC experiments in 1988. Quite the contrary, much smaller fractions, even compatible with zero, were measured ($\Delta\Sigma = 0.12 \pm 0.17$, European Muon Collaboration). The quark thesis of fermionic spin-$\frac{1}{2}$ particles was not experimentally confirmed. Here, from a scientific perspective, one should have argumentatively examined the “quark idea” – based on experiments. This missed (possible) turning point cannot be “stressed” enough. With what justification are quarks (even today) “presented” as spin-$\frac{1}{2}$ particles?

Psychologically, this is easy to understand, since the SM had already existed for more than 20 years at the end of the 1980s. The quark–parton model (QPM) developed by Richard Feynman in the 1960s describes nucleons as composed of fundamental point-like components, which Feynman called partons. These components were then identified with the quarks postulated a few years earlier simultaneously by Gell-Mann and Zweig. Thus, at the end of the 1980s, one hoped that future experiments would confirm the half-integer spin of the quarks. An abandonment of the hypothesis that quarks are fermions would have been the end of the SM. Then something phenomenologically strange “happened”. Further thought approaches and experiments brought new theoretical elements into play. Strange in the sense that with this idea the quark as carrier of an intrinsic spin contribution, on closer inspection, was already conceptually abolished.

25.6.2 For (Better) Understanding, Some Background on the Spin Problem

The quantum mechanical spin has “existed” at least since 1930, inconsistent in consideration and without “spin phenomenology”, purely mathematically, see for example:

Chapter 10.2 Dirac’s electron theory 1928, page 10006: “For the new angular momentum has nothing more in common with what one can imagine under this name as a mechanical quantity. It arises from no motion, but from the interaction of a spatial vector with the Dirac matrices in the space of their four abstract dimensions."

In other words: Quantum mechanics (QM) often "works" in illustrations and semantic explanations with a false suggestion using the term spin (intrinsic rotation), but the associated QM formalism describes no such real-physical rotation. "Simply" put: The quantum mechanical spin has nothing to do with rotation and is, so to speak, nothing more than a necessary but completely unfounded (i.e., without real-physical intuition) quantum number that is generated purely mathematically within the framework of prevailing physics (four-component Dirac spinor field with four Dirac matrices).

To prevent possible misinterpretations (of "non-SM initiates"), it should be noted that both the magnitude of the spin value of the elementary particle "quark" required by the theory and the resulting magnitude of the spin value of the proton composed of 3 quarks, according to the SM, should be $\frac{1}{2}$.

Werner Heisenberg concluded in 1932 that proton and neutron are two states of one and the same particle, the nucleon. To distinguish them, isospin was introduced. The z-component of the isospin of the neutron is $-\frac{1}{2}$, that of the proton $+\frac{1}{2}$. In the quark model, these stand for the two quarks up quark ($I_z = \frac{1}{2}$) and down quark ($I_z = -\frac{1}{2}$). For antiquarks, the sign of I_z changes. The quarks s (strange), c (charm), b (beauty), and t (top) carry no isospin.

Isospin is a property of all elementary particles with strong interaction. The photon and the leptons (electron, muon, neutrino) have no isospin. The property "isospin" cannot be represented by a simple numerical value, but is a quantized quantity with three components that behave like the components of a vector. However, this is not a vector in ordinary physical space, but a vector in an abstract "isospin space" or "I-space". Since all elementary particles and systems made of them also carry a characteristic real quantum vector with them, their "spin", a formal analogy of isospin with this spin arises. Beyond this formal relationship, isospin has nothing to do with spin.

More generally: Quantized properties are characterized here by internal symmetries and have nothing more in common with properties in the usual sense that can be conceived as physical qualities inherent in things. The isospin of nucleons or the "color" of quarks no longer express any qualities in this sense at all, but only arbitrarily fixed basis states or directions in an abstract space that are related to each other by symmetry transformations. But even the resulting assumption that the gluons brought into play contribute to the proton spin did not yield the desired result. In the third, currently current version of the theory, quarks, gluons, and their dynamic-relativistic orbital angular momenta are supposed to neatly make up the proton spin in the result.

On closer inspection, this second post-correction has the "advantage" that the result is "calculated" purely algorithmically in large computer systems within the framework of lattice gauge field theory and constructs such as "pion clouds", and from the perspective of SM believers cannot be falsified. Thus one "combines" until the desired result is iteratively obtained.

But here comes the huge, overpowering, logically rationally founded BUT: this measure obviously does not justify classifying quarks as fermions. For no matter how constructed the asymmetric ensemble of unobservable postulated theoretical objects and interactions is announced, the quarks themselves do not thereby become spin-$\frac{1}{2}$ particle entities.

The state of affairs can (fortunately) be formulated generally. The claim that an entity (quark) possesses a measurable intrinsic value is refuted when measurements provide no such results. The addition of further postulated theoretical objects (postulated gluons) and further postulated interactions (relativistic effects) may, in the sum of events, yield the theory-desired value, but thereby the original entity does not receive or contain the postulated intrinsic value. This argument is as simple as it is understandable.

And now? Without quarks as fermions, the SM is at an end. This circumstance is not addressed, suppressed, and if necessary concealed. It is naive to believe that the rationally-logically understandable insight "quarks are not fermions" will "simply" prevail. For then all previous SM efforts are "history". Epicycle theory and phlogiston theory send their regards. It requires publicly perceptible "social" situations that, with experienced and competent moderators, find wide dissemination in order to put an end to the fake news of the SM, known for decades, which corresponds to a super-GAU of epistemology and a super-GAU of logic.

(An example in passing: Ms. Hossenfelder has, despite the fact that she is in part a critic of the SM, not published my comments on this in her topic-relevant blog posts. A possible reason: Ms. Hossenfelder "likes" quarks as fermions – or more precisely: Ms. Hossenfelder also needs quarks with regard to the quantum gravity she favors, since without fermionic quarks the SM and ultimately every associated current quantum field theory variation is void for lack of a fermion basis.)

25.6.3 Quark–Parton Model and Deep Inelastic Scattering

The quark–parton model (QPM) developed by Richard Feynman in the 1960s describes nucleons as composed of fundamental point-like components, which Feynman called partons. These components were then identified with the quarks postulated a few years earlier simultaneously by Gell-Mann and George Zweig. According to the quark–parton model, a deep inelastic scattering event (DIS) is to be understood as an incoherent superposition of elastic lepton–parton scattering processes. This picture, however, only holds if the momentum transfer by the photon is sufficiently large so that the individual partons can be resolved; i.e., one is in the region of lepton–nucleon scattering referred to as deep inelastic.

A cascade of interaction conjectures, approximations, corrections, and additional theoretical objects "refined" the theoretical nucleon model in the following period. Among other things, nucleon structure functions result as summations of the parton structure functions over postulated helicity and charge states of all "quark flavors" postulated in the nucleon. In the quantum-chromodynamics-extended parton model, quarks are supposed to radiate gluons that are either reabsorbed by the quarks themselves, or produce quark–antiquark pairs, or radiate further gluons. These "secondary partons" form a "cloud" around the original quark. Within the framework of this model, a quark is no longer a point-like object, as in the original QPM. Without going into the hodgepodge of further assumptions and resulting supposed equations of the SM for the extended postulated substructure of nucleons, a clear statement follows:

A fundamental (epistemological) problem is immediately recognizable. All experimental setups, implementations, and interpretations of deep inelastic scattering are extremely strongly theory-laden. For clarification of this statement, see, for example, Chapter 2.1 Kinematics of deep inelastic scattering of the explanations on COMPASS experiments. (The COMPASS experiment, which began operation at the European nuclear research center CERN in Geneva/Switzerland in 2001, investigates the spin structure of the nucleon in a broad program by means of deep inelastic scattering.)

An example of a relatively recent "theoretical attempt" to arrange free parameters and arbitrary theoretical objects so that what is known comes out at the end: Spin Crisis of Proton and Baryon's Magnetic Moment (DP Tiwari and RS Gupta 2015).

Moreover, the postulated asymmetry of the quark charge carriers in the form of strongly differing mass(-limits) and differently fragmented electric elementary charge is an epistemological imposition, especially under the aspect that quarks cannot be detected (confinement thesis). These people are so convinced of their belief that they have apparently lost sight of the essential. Why should a complex, multi-object-asymmetric, charge-fragmented, dynamic substructure create a spin value $\frac{1}{2}$ and a whole elementary charge e via dynamic states in the temporal or statistical average? The comparison with the SM-postulated point-depleted, "leptonic" electron, with spin value $\frac{1}{2}$ and whole elementary charge e, which accomplishes this without "dynamic effort" and structure, identifies the quark–gluon thesis as a belief fairy tale. This fairy tale should appear as such even to people of average intelligence.

As a reminder: Quarks are not particles, neither in the phenomenological nor in the quantum theoretical sense, since they do not occur as isolatable particles or states. Physical particles, on the other hand, are to be thought of as bound states composed of quarks. No physical objects correspond to the elementary quantities of quantum field theory. Thus the desired, different kinds of postulated elementary particles in the SM differ by the quantum numbers of dynamic properties such as charge or isospin. Some are massless by postulate, others not. Electrons are theory-desired impoverished to mass and charge points. Some others should be massless, like neutrinos, but then are not. Occurring mathematical theory fragments, such as "5 phases" in the CKM matrix, are simply discarded because they do not "fit" in a result-oriented way. It says laconically on the topic of "quark mixings": "The CKM matrix (Cabibbo–Kobayashi–Maskawa matrix) is physically uniquely described by three real parameters and one complex phase (five further phases that occur mathematically have no physical meaning)". This simply means that one takes, result-oriented, the mathematical elements that "somehow fit" and simply ignores others. This arbitrary procedure within the framework of mathematical models has nothing more to do with exact science.

Nevertheless, quantized properties are characterized by internal symmetries and have nothing more in common with properties in the usual sense that can be conceived as physical qualities inherent in things. The isospin of nucleons or the "color" of quarks no longer express any qualities in this sense at all, but only arbitrarily fixed basis states or directions in an abstract space that are related to each other by symmetry transformations. Almost all previously known symbol systems are cited. Sometimes it is colors (red, blue, green), sometimes letters (u, d, s, c, b, t), sometimes symbolic properties (strange, charm, beauty, . . .), and as a term the flavors are also added; for a structure lying even below the quarks, the designations "tohu" and "wabohu" from the creation story in the Old Testament were proposed. Among other things, religiosity, "neurological abnormalities", megalomania, as well as science-freed "kindergarten language" come into "play" with the theorist as a "confused", (mathematics-)believing person.

Ernst Mach already noted: "Whoever does mathematics can sometimes be overcome by the uneasy feeling that his science, indeed his pen, surpasses him in cleverness, an impression that even the great Euler, by his own admission, could not always resist." [EM1]

[EM1] Ernst Mach (1838–1916), lecture, session of the Imperial Academy of Sciences in Vienna on May 25, 1882.

25.7 Heavy Ions in Plasmatic Dream States – a Pathetic Attempt to Practically Circumvent the Confinement Thesis

The LHC is designed to accelerate protons to energies of 7 TeV and thus enable collisions with a center-of-mass energy of 14 TeV. In the range of accelerated heavy ions, such as lead nuclei, the LHC reaches energies of 2.7 TeV per nucleon; in collisions of lead nuclei with 208 nucleons, collisions at an energy of 1150 TeV theoretically take place. The currently second-strongest ion accelerator is the Relativistic Heavy Ion Collider (RHIC) in Brookhaven (USA), which reaches a theoretical maximum energy of about 78 TeV in collisions of gold nuclei at 0.2 TeV per nucleon. The ALICE experiment is designed to evaluate these events, with which the postulated quark–gluon plasma is to be investigated.

So much for the theory and practice of the Standard Model. But the postulated quark–gluon plasma detections are not direct experimental proofs, but strongly theory-laden interpretations of experimental results. This also applies to heavy ion collisions. The actual communication problem begins much earlier. The term "plasma" is wrong with respect to the theoretical objects quark and gluon.

The quark masses postulated according to the SM do not by far add up to the nucleon masses. Gluons are massless.

- Postulated up quark mass: $2.3 \pm 0.7 \pm 0.5$ MeV/c^2 up (u)
- Postulated down quark mass: $4.8 \pm 0.5 \pm 0.3$ MeV/c^2 down (d)
- 938.2720813(58) MeV/c^2 proton mass (duu) $\sim$ 0.8–1.2 % (!!!) quark mass fraction
- 939.5654133(58) MeV/c^2 neutron mass (ddu) $\sim$ 1.1–1.4 % (!!!) quark mass fraction

Thus heavy ions composed of protons and neutrons (such as lead or gold nuclei) cannot be represented by quarks and gluons either. This means that, according to the mass–energy equivalence principle, nucleons and ultimately heavy ions consist almost entirely of phenomenologically indeterminate binding energy. Even more serious is the fact that the heavy ions are accelerated to nearly the speed of light before they collide. This means that in addition to the binding energy, a considerable amount of external energy is added. How this phenomenologically imaginably divides into translational energy and "mass equivalence" is not told to us by relativity theory.

Richard Feynman's statement "It is important to realize that in physics today, we have no knowledge of what energy is." has not changed to this day. Physics lives from secondary quantities or secondary concepts that all possess no (consistent) phenomenology.

More precisely: If 2.7 TeV per nucleon at the LHC falls to lead ions, then with a mass fraction of the quarks of about 1 %, the percentage mass fraction of the event is just 0.00035 %.

Conclusion: Not according to an opinion, but according to the state of knowledge and the postulates of the SM, in particle collisions mainly phenomenologically indeterminate accelerated energy packets, i.e., binding energies, and mainly external energy collide with each other; of elementary particle masses there is practically no "trace".

One can obviously "explain away" contradictions with observation in any theory by incorporating further (unverifiable) postulates into the theory that precisely "explain" this observation.

25.8 Brigitte Falkenburg's Analysis: Particle Metaphysics

Brigitte Falkenburg writes in *Particle Metaphysics: A Critical Account of Subatomic Reality* (2007), among other things:

> "It must be made transparent step by step what physicists themselves regard as the empirical basis for today's knowledge of particle physics. And it must be transparent what they concretely mean when they speak of subatomic particles and fields. The continued use of these concepts in quantum physics leads to serious semantic problems. Modern particle physics is indeed the hardest case for incommensurability in Kuhn's sense."

> "Finally, theory dependence is a poor criterion for distinguishing between secure background knowledge and uncertain assumptions or hypotheses."

> "Subatomic structure does not really exist in itself. It only shows itself in a scattering experiment with a certain energy, i.e., due to an interaction. The higher the energy transfer in the interaction, the smaller the measured structures. In addition, according to the laws of quantum field theory, new structures arise at very high scattering energies. Quantum chromodynamics (i.e., the quantum field theory of the strong interaction) states that the higher the scattering energy, the more quark–antiquark pairs and gluons arise inside the nucleon. According to the model of scattering in this region, this in turn leads to scaling violations that have actually been observed. This sheds new light on Eddington's old question of whether the experimental method leads to discovery or to production. Does the interaction at a certain scattering energy reveal the measured structures or does it create them?"

> "It is not possible to trace a measured cross section back to its individual cause. No causal story relates a measured form factor or structure function to its cause."

> "With the beams produced in particle accelerators, one can neither look into the atom, nor see subatomic structures, nor observe point-like structures inside the nucleon. Such talk is metaphorical. The only thing that makes a particle visible is the macroscopic structure of the target."

> "Niels Bohr's quantum philosophy... Bohr's claim was that classical language is indispensable. This remains valid to this day. At the individual level of clicks in particle detectors and particle tracks on photographs, all measurement results must be expressed in classical terms. The use of the known physical quantities length, time, mass, and momentum-energy on the subatomic level is indeed due to an extrapolation of the language of classical physics into the non-classical realm."

Brigitte Falkenburg clearly shows that the experimental side of the SM also measures no analytically calculated matter within the framework of a real physical experiment. Ununderstood binding energy and kinetic energy collide here.

In modern theoretical model physics, on closer inspection, it is about preserving existing thought dogmas. Arguments of reason are sacrificed to habits of thought and Standard Model-laden prejudices. A psychological reason lies in belief and hope. The majority of "researching" physicists are without exception adherents of quantum field theories. An understanding of these opaque theories is not possible. Building on the ununderstood works of others, they plan experiments for which there are only two results: success and confirmation, or failure and brief perplexity, which then usually ends in new particles, new quantum numbers, and theory extensions.

Like all believers, the (theory-affine) experimenters also tend to consciously and unconsciously avoid failure and instinctively evade a refutation of their theory-laden belief by an experiment (or by thought). Instead of limiting experiments to the essential and making them as simple and efficient as possible, attention is often diverted from the essential. Modern theories based on quantum field theories all have one thing in common: In theory-relevant experiments, "sooner or later" they turn out to be trick packages. Instead of discarding the theory, merely theory corrections are made. These can be, for example, postulates about new virtual particles that "briefly" violate the law of energy conservation with reference to Heisenberg's uncertainty relation – although this violation ignores the most important conservation law of system physics.

In this context, it is astonishing how many "interested parties" and "science professionals" firmly believe that quantum field theories are experimentally verifiable, innovative theoretical concepts that generate(d) practical applications.

25.9 Historical "Fun Fact" with Far-Reaching Consequences

When Peter Higgs (1929–2024) presented "his" Higgs mechanism in 1964, he was in the "LSD flower power era", whose contemporary scientific zeitgeist evaluation was shaped by protagonists like the philosopher of science Paul Feyerabend (1924–1994), whose idea of science was: "Anything goes". This possibly explains why Higgs started his, from the Standard Model perspective "groundbreaking", explanations with unreal tachyons.

[Tachyons (ancient Greek $\tau\alpha\chi\acute{\upsilon}\varsigma$ tachýs "fast") are hypothetical particles that move faster than light (superluminal). There is obviously no experimental evidence that such particles exist.]

Higgs's irrationality was welcomed at the time; Peter Higgs was, so to speak, a science avant-gardist in Feyerabend's sense. Feyerabend fits with "Feierabend" (end of work) with the meaning: the actual work is finished. In retrospect, Peter Higgs created and lived in the scientific end of work – meaning: phenomenologically, rationally founded thought-model physics work no longer took place from then on.

The quark–parton model (QPM) developed by Richard Feynman (1918–1988) in the 1960s also describes nucleons as composed of fundamental point-like components, which Feynman called partons. These components were then identified with the quarks postulated a few years earlier simultaneously by Gell-Mann (1929–2019) and George Zweig (born 1937).

Here too the foundation is irrational, because the resulting mathematical approach of the Standard Model of particle physics (SM), starting from zero-dimensional, massless objects, obviously provides no connection to the perceptible physical reality in which mass and extension represent fundamental properties. Massless gauge bosons move at the speed of light according to the formalism requirement. The SM describes matter formation and interactions through purely abstract mathematical symmetries (gauge symmetries with their gauge groups).

25.10 Parametric Creeds

"Modern" elementary particle physics "fed" on "speculation cascades". In no other scientific discipline is there such "creative" postulating, "discovering", and "modifying" as in particle physics. That this construct of particles and theses successfully fertilizes itself and confirms itself within its own boundaries is no great surprise, since the activists can at any time introduce new particles and theses to "save" "old" theses and particles.

25.10.1 Back to the Higgs Mechanism

The following state of affairs is hardly addressed or "partially concealed": The Higgs mechanism starts with a tachyon field and thus inherently with a negative mass squared ($m^2 < 0$). Note: The original Higgs field is a tachyon field, mathematically definable, physically unreal. To "circumvent" the tachyon term, the field is reparameterized as a variation around a vacuum state. This changes the sign of the mass term.

The required properties of the Higgs field include:

- Higgs particles are bosons, because only then is a coherent wave function possible.
- For the same reason, Higgs particles must interact with each other.
- The Higgs field is scalar in order to preserve the symmetry of the vacuum.
- In the present case of the $U(1)_{em}$ gauge theory, the field must be charged in order to couple to the photon.
- The Higgs field satisfies the Klein–Gordon equation:

$$(\Box + m^2)\Psi = 0$$

  where $\Box$ is the d'Alembert operator (Lorentz scalar, invariant under Lorentz transformations).

- One postulates that the Higgs field is in the ground state with a vacuum expectation value $|\Psi_0|^2 = \text{const} \neq 0$.
- The requirement of a non-vanishing expectation value in the ground state is not trivial and can only be "fulfilled" with a self-interaction of the field.

In order to realize mass generation in the Standard Model through the Higgs mechanism, as a minimal variant one can set the Higgs field as an isospin doublet. In the course of this mathematical procedure, it turns out that another massless vector boson, the so-called Goldstone boson, occurs. However, since there is no experimental indication of this boson, it is declared "unphysical" and mathematically eliminated ("gauged away").

Furthermore, it should be considered: The Higgs potential and thus the spontaneous symmetry breaking of the electroweak symmetry is added "by hand" to the SM. There is no dynamic explanation for this mechanism. The four components of the isospin doublet approach "provide" ultimately three vector bosons with a longitudinal spin component and one scalar Higgs field. So that the photon remains massless, in this case the vacuum expectation value is (again) "set" to zero. The "remaining" mass terms are then generated by the "neutral" component with non-vanishing expectation value. After (mathematical) "rearrangement", the gauge-invariant $SU(2) \times U(1)$-Lagrangian density results with the "desired" mass terms for the Higgs and gauge bosons.

Generally noticeable: Fundamentally, mass and masslessness of the theoretical objects are assigned according to the fantasy wishes of the theory builders. "Fantasy wish" is not a semantic barb, but the correct word in content. For upon neutral consideration of the SM, it is noticeable that within the framework of the formalism mass and masslessness are assigned according to the discretion of the theory builders. This can fundamentally be realized by “re-gaugings”. But this arbitrary action possesses no real-physical nutritional value in the sense of gaining knowledge about phenomenological connections. Quite the contrary: A physical pseudo-reality is created that then serves as a binding basis. How senseless this undertaking is is shown exemplarily by the theoretical demand for masslessness of the neutrinos, which, however – according to (recognized) neutrino oscillations – possess finite masses. The theoretical demand comes to nothing, because the neutral leptons (neutrinos) are assumed in the Standard Model as exactly massless Weyl fermions. The contradiction to experimentally required neutrino masses is evident.

Here the circle of fantastic arbitrariness closes in the direction of the cosmological standard model.

25.11 Standard Thought Model Anatomy – “Prelude Thoughts”

[MF] This leads, among other things, to the following, extremely important answer to the question:

What remains of General Relativity (GR) or the resulting Standard Model of cosmology (ΛCDM model)?

The English Wikipedia points out, unlike the German version, the essential aspect of the model:

> “The Lambda-CDM, Lambda Cold Dark Matter or ΛCDM model is a **mathematical model** of the Big Bang theory.”

> “The Lambda-CDM, Lambda cold dark matter or ΛCDM model is a **mathematical model** of the Big Bang theory.”

Overall, as an interested person, one must ask why the German Wikipedia, compared to the English Wikipedia, has almost “nothing to say” about the ΛCDM model. German speakers, unless they also consult the English version, remain “knowledge-technically” left out.

Obviously almost no one is clear, whether layperson or theoretical physicist, what “mathematical model” means here. Put succinctly, it means: The only “true” thing in this model is differential geometry. Everything else in the model are speculations, empirically unfounded postulates, and all of this “everything” built on free parameters.

The history of the Big Bang theory proceeded in such a way – without going into detail here – that new hypothetical “objects” had to be introduced for which there was and still is no evidence, except that they save(d) the underlying theory. The term “inflation”, which includes a field and an energy that are completely unknown, was introduced in the early 1980s to keep the Big Bang despite very serious contradictions in observation. Soon after, non-baryonic “dark matter” was added as a postulated theoretical entity, and in 1998 “dark energy”. Dark energy is a postulated, non-detectable form of energy that influences the universe on large scales. Its postulated main effect is that it triggers or maintains the accelerated expansion of the universe. The term was coined in 1998 by the astrophysicist Michael S. Turner.

Among other things, due to the necessary thesis for maintaining the ΛCDM model, the inflation phase (with multiple superluminal velocity) was introduced; for this, space had to be decoupled from matter without explicitly mentioning it. “Implicitly” this is clear, because matter, even within the framework of the standard models, cannot be accelerated to the speed of light, let alone to superluminal speed. This means, however, that this space–matter decoupling also existed before and after the postulated cosmic inflation, i.e., also in the here and now. For this assumption there was and is no evidence.

25.11.1 Cosmological Constant Λ and Postulated Inflation

The cosmological constant Λ, which Albert Einstein introduced in the belief in a static universe and which was then abolished in the course of the "early ideas" of the expanding universe, came back because Λ was theoretically needed again. But the cosmological constant simply means that, even in the understanding of Big Bang theorists, the "vacuum" possesses an energy density. Vacuum here is another wordplay for supposedly empty space. Attaching Λ to the theory, however, inevitably leads to "matter in space". The result is easy to understand. Let us leave aside for a moment that we do not want to address the attribute of light-speed propagation in a nitpicky way here. Space expands with matter (nearly light-speed) until the beginning of the postulated superluminal inflation, then space expands without matter until the end of inflation, and then again expands with matter well-behaved within the framework of relativity theory.

More detailed: Inflationary models assume that the vacuum state of the universe about 10^{-36} seconds after the Big Bang was different from today: The inflationary vacuum had a much higher energy density. According to General Relativity, every vacuum state with an energy density not equal to zero generates a repulsive force that leads to an expansion of space. In inflationary models, the early high-energy vacuum state thus causes extremely fast expansion, i.e., an expansion with multiple light speed. This expansion explains various properties of today's universe that cannot be explained without such an inflationary epoch. In most inflation models, a scalar field, the so-called inflation field, is proposed that possesses the necessary properties. The inflation theories leave open when the inflation epoch ended; currently one works with a duration between 10^{-33} and 10^{-32} seconds after the equally postulated Big Bang. The rapid expansion of space in the model has the consequence that all potential elementary particles (or other "undesirable" artifacts such as topological defects, whatever that may be) that were "left over" from the time before inflation were now distributed "very thinly" across the universe. In the postulated inflation phase, the linear dimensions of the early universe increase by a factor of at least 10^{26} (and possibly by a much larger factor; the theories leave this open) and the universe increased its volume by a factor of at least 10^{78}. For comparison: An expansion by a factor of 10^{26} corresponds to the expansion of an object from 1 nanometer to a length of about 10.6 light-years – postulated in 10^{-33} to 10^{-32} seconds.

25.11.2 Inflation Conclusion

It doesn't get more fantastic and arbitrary in well-known science fiction films like Star Wars. The difference: These are theoretical physicists who, according to the Pippi Longstocking principle, make the world as they like it. Why? Because otherwise – i.e., without an ultra-short, superluminal inflation phase, for which, by the way, there is no phenomenological justification within the framework of inflation theory – the self-defined theory would refute itself due to then inexplicable observation interpretations and, if you will, associated axiomatically. What one, especially as a layperson, would not expect and would most likely assess as dubious, if one could understand it. Nothing remains of either standard model. Neither generally, nor epistemologically, or in terms of complexity and arbitrary complexity.

25.11.3 Considered at a Higher Level

It should astonish not only interested parties, but especially well-trained scientists, like astrophysicists, that they think, more precisely are certain, that they can make statements about cosmic processes outside our solar system. On closer inspection, this "astrophysicist thinking" is not only improbable but absurd. In all cosmological "observational studies" we are not dealing with controllable laboratory experiments. The causes and phenomena of all possible object observations outside our solar system cannot (simply) be assumed as known. Furthermore: The human observation time span is extremely small compared to the time spans in which cosmic motions took place and take place. To base assumptions on the data from the human observation duration is “far-fetched”, to put it casually. All current supposedly empirical measurements are strongly (Big Bang) theory-laden. Postulated time spans, distances, and energy densities are subjective and theory-dependent.

25.12 As a Reminder: The Basis-Violated Covariance Principle

General Relativity is a theory of gravitation and assumes the equality of inertial and gravitational mass [“equivalence principle”]. But GR does not explain this principle; it presupposes it. General Relativity was born, among other things, from the demand to be able to use arbitrary coordinate systems for describing the laws of nature. According to the covariance principle, the form of the laws of nature should not depend decisively on the choice of the particular coordinate system. This demand is causally mathematical and leads to a variety of possible coordinate systems [metrics], but remains physically “unmotivated”. Worse: Depending on the choice of coordinate system, phenomenological interpretive leeways arise (see above). The reason is “relatively” simple: Coordinate systems are mathematical constructs that (need) possess no physical properties.

The equation systems (Einstein, Friedmann) of General Relativity, which underlie the statements of the Standard Model of cosmology, provide no analytical solutions. Only idealizations and approximations lead, to a limited extent, to computable solutions. The unavoidable (“covariant”) contradictions come with the obviously inadmissible idealizations and approximations of the system of nonlinear, coupled differential equations.

Mathematically, the covariance principle cannot be “violated”, since it is axiomatically founded. Only this axiomatic precondition “disappears with the mutilation” (idealization and approximation) of the actual equations. In other words: The mathematically correct equations have no analytical solutions. The reduced equations (approximations, idealization) do have solutions, but these are not covariant. Thus no solution has a real-physically founded meaning. In other words: The covariance principle, viewed real-physically, is formal “blah blah” and can produce any theoretical crap. Even typographical errors can generate solutions within the framework of the differential geometry of GR.

As an exemplary thought impulse: The Eddington–Finkelstein coordinate transformation, named after its authors for the “Schwarzschild metric” solution possibility, eliminates the coordinate singularity of the Schwarzschild solution and “ensures” that for the “advanced” solution inward and for the “retarded” solution outward particles can enter and exit the black hole! In other words: The postulated black holes of the “original version” of the Schwarzschild metric were, on closer inspection, the result of two integration constants of the chosen coordinate system. Another coordinate system by Messrs. Eddington and Finkelstein fixes the coordinate artifact, but “gives” the supposedly black hole the property that particles can leave the black hole. In effect: No black hole.

Conclusion: Black holes are, popular-scientifically, unquestionably more sensational than no black holes. But there is no experimental proof of the existence of even a single black hole. Black holes are nothing more than theoretical objects of a mathematical formalism whose required covariance principle cannot be fulfilled due to the structure of the equation systems, since only approximations lead to computable solutions. Black holes are popular, as Theodor Fontane noted: “the sensational counts and only to one does the crowd stream even more enthusiastically, to bare nonsense.”

25.13 Overall Assessment

Whoever studies the history of elementary particle physics somewhat neutrally will hardly be able to believe that these are natural scientific considerations and efforts in the sense of a purposeful simplification and unification. Whenever experimental physics refuted the theory(ies), the theory was extended by means of new elementary particles, renewed substructuring, and if necessary new quantum numbers that "handled" the missing properties, the missing energy, or the missing symmetry. This has little to do with science. Instead of tackling the problems with a new radical (theory) approach, small and large cosmetic corrections are continuously carried out. Instead of simplifying and unifying, it is "mercilessly" extended and "specialized". A hodgepodge of particles (properties) emerges. For every case of an undesired deviation, for every particle, theory-preserving special regulations are added as needed.

The mathematical formalism is the smallest problem in this fantasy process. If necessary, divergent terms are regularized, renormalized, or simply declared "unphysical", simply "omitted". That this destroys the axiomatic basic structure seems to be clear to only a few, or is obviously irrelevant to the majority of theorists. Are they ultimately only – from a natural scientific perspective unacceptable – economic interests of the beneficiaries and lobbyists who skim research funds and want to justify their material existence?

If one relates the enormous number of publications in the area of the standard models to the epistemological gain, it is – casually formulated in our current digitalized age – almost exclusively data garbage. This is simply because the supposed measurement results are already strongly theory-laden, and the theoretical approaches stem from idealized, approximated equations that in their original form possess no formal-analytical solutions and whose approximation-reduced designs provide nothing more than mathematical possibilities that are real-physically meaningless, especially since, via the feedback of theory-laden selective measurements, only the desired measurement results are further used. Thus a knowledge-freed, formally and economically self-sustaining system arose and develops.

That the system developers and profiteers want to eliminate any halfway logical alternative research from the outset is egoistically humanly understandable. Every formal-analytical solution that delivers real-physical values represents an existential threat to the standard models. Popular-scientifically, the system is carried by servile science magazines, their indoctrinated readers, and unknowing epigones who applaud every popular theory nonsense. The suspicion arises that people whose creative drive cannot be satisfied in a suitable way, because there is really nothing new for them to find, document and formalize trivialities, clothe them in new words, and use sentence structures that a normal person does not understand and whose respect the author quasi-automatically captures.

In the end, it makes no difference whether there are scientists in this system who "really" believe with full fervor in these "theoretical implications", which are hard to surpass in naivety and contradictions, or whether it is ultimately banal economic interests paired with academic power that scientists exploit as elite beneficiaries in order to celebrate and proclaim bare nonsense.

Particularly egregious is the circumstance that, compared to organized religions, theoretical "basic particle physics" claims to embody a high degree of objectivity and freedom from belief. And that is how the interested public sees it too. Theoretical physics is certainly in many minds one of the last arenas where one suspects creeds instead of science.

25.14 Church and Physics

Kurt Marti shows fragmentarily how the Vatican and CERN researchers hand in hand imagine the faith future of particle physics and cosmology.

> "The Vatican is not primarily concerned with elementary particles and black holes, but with metaphysics. With their excellent sense for the seemingly essential, the theologians in Rome have long understood that particle and astrophysics is operating more and more in the thin air of experiment-free metaphysics."

See Elective affinities of CERN and the Vatican by Kurt Marti.

> "At the summit of power and recognition, people suddenly become talkative, coquet with their tricks and small cheats, mock ethics and morality, and boast of their ability to have pushed through a very special personal interest with the help of manipulation and clever propaganda. Many a suspicion about the true nature of a successful person finds confirmation through such vain self-revelation, but is by no means able to shake the person's position of power. Established is established." [1]

25.14.1 Euphemistic Message of the German Physical Society (DPG)

> "The German Physical Society shall serve exclusively and directly pure and applied physics. . . . "

25.14.2 "True Message" of the German Physical Society (2000)

> "Big Bang cosmology is, in a sense, the modern, physical version of the creation story." [2]

[1] When the Big Bang was fashionable – memories of a curious world model, Klaus Gebler 2005, ISBN 3-8334-3983-1, online

[2] A comprehensive memorandum "PHYSICS – Topics, Significance and Perspectives of Physical Research, a Report to Society, Politics and Industry" distributed to all secondary schools in 2000 contained this memorable sentence: "Big Bang cosmology is, in a sense, the modern, physical version of the creation story. . . . "

Klaus Gebler makes clear in his remarkable book *When the Big Bang was fashionable – memories of a curious world model*, among other things, how much "modern physics" is influenced by the church's view. Website: Klaus Gebler

Didactically, the prevailing epistemological emptiness has been a fixed component of education for decades. Thought monotheism is prescribed popular-scientifically even to schoolchildren. Should these, despite the immense indoctrination and against all expectation, one day realize the quark-based proton fairy tale, they stand completely alone. Who believes that tens of thousands of scientists over several generations occupied themselves with something if it does not represent the non plus ultra? "Billion-dollar" particle accelerators were and are not built if all of that is nonsense? These "psychological components" weigh heavily.

26 The Logical-Argumentative Refutation of LIGO and Gravitational Waves: A Methodological and Epistemological Deconstruction

The claim that the Laser Interferometer Gravitational-Wave Observatory (LIGO) has *directly detected* gravitational waves is subjected to a methodological, theoretical, and epistemological examination. The analysis shows that the LIGO "detections" are the result of a highly theory-laden data analysis based on non-covariant approximations of General Relativity (GR). The claimed measurement accuracy of 10^{-22} exceeds the methodologically controllable accuracy by ten orders of magnitude. The underlying gravitational wave solutions violate the covariance principle of GR and are embedded in a Newtonian space container – a logical contradiction to the axioms of the theory. The work closes with a sociological analysis of the immunization strategies of the ΛCDM model and documents the systematic suppression of criticism by the established scientific community.

26.1 Introduction

The discovery of gravitational waves by the LIGO collaboration was celebrated as a breakthrough of the century. The present work undertakes a systematic deconstruction of this claim. It shows that the LIGO "detections" do not hold what they promise: a direct, empirical proof of gravitational waves. Instead, they are the result of a highly theory-laden data analysis based on methodologically uncontrollable assumptions.

The argumentation is divided into five levels:

1. The methodological impossibility of the claimed measurement.

2. The theoretical inconsistency of the gravitational wave solutions.

3. The epistemological impossibility of a "spacetime measurement".

4. The historical uncertainty regarding the existence of gravitational waves.

5. The sociological immunization of the ΛCDM model.

26.2 Level 1: The Methodological Impossibility of the Claim

26.2.1 The Discrepancy of the Order of Magnitude

The most precise laboratory measurements in physics (e.g., of the magnetic moment of the electron) achieve a relative accuracy of about 10^{-12} [1]. LIGO claims to measure length changes of 10^{-18} m over a 4 km long distance – that corresponds to a relative accuracy of 10^{-22} [2]. This claimed accuracy is **ten orders of magnitude** higher than anything ever achieved under controlled laboratory conditions.

The spacetime is not sensually experienceable and also not instrumentally measurable. Spacetime is a mathematical construct.

In all cosmological "observational studies", we are **not** dealing with controllable laboratory experiments. The human observation time span is extremely small compared to the time spans in which cosmic motions took place and take place. To base assumptions on the data from the human observation duration is "far-fetched", to put it casually. All current supposedly empirical measurements are strongly (Big Bang) theory-laden. Postulated time spans, distances, and energy densities are subjective and theory-dependent.

It would be naive and foolish to ignore widespread expectations. It is not easy to explain to people who expect at least in the core of a matter a proximity to reality and proportionality that this is often not the case, especially when it concerns scientific topics. Particularly egregious is the circumstance that, compared to organized religions, theoretical "basic physics" including associated theory-laden experimental physics suggests embodying a high degree of objectivity and freedom from belief. And that is how the interested public sees it. Physics is certainly in many minds one of the last arenas where one suspects creeds instead of science.

26.2.2 Dissemination Strategy of Object and Origin Myths

It basically starts "properly", see for example the YouTube video Simulation of the neutron star coalescence GW170817 [11] The description by the Max Planck Institute for Gravitational Physics (Albert Einstein Institute) begins with ... "The video shows a *numerical* simulation"...

But none of the proclaimers, whether scientists, science journalists, news anchors, ..., ultimately "means" that it is, both theoretically and physically, nothing more than hypotheses and simulations. Strongly theory-laden wishes are "materialized in good (ambiguous) material faith". Although everyone could see what he will never really see...

26.2.3 Perception Possibilities

In our solar system there are neither neutron stars, gamma ray bursts (GRBs), nor black holes (respectively "anomalies" that can be interpreted as such).

A list of postulated "nearest" black hole candidates can be found at `https://en.wikipedia.org/wiki/List_of_nearest_black_holes` with a "shortest" distance of 2800 light-years. For comparison: The nearest star "from our perspective" is Proxima Centauri at 4.24 light-years (`https://de.wikipedia.org/wiki/Proxima_Centauri`). Object and distance data refer to the "perspective of the ΛCDM model".

The situated sociological perception problem "consists" in the fact that here, according to a simple psychological belief pattern, diverse postulated theoretical objects of the most varied kinds, some for decades – to the population equipped with rudimentary knowledge – are literally "sold" as 100 % really existing.

Space and time are primarily "ordering patterns of the understanding". In order to "obtain" physics from these ordering patterns, a phenomenological consideration and explanation is mandatory.

An example is *spacetime*. The postulate states that within the framework of the (formalized) consideration of relativity theory, time and space exist only as a unity. A consequence: We cannot capture the events in the postulated general-relativistic cosmos temporally as a snapshot, because time does not exist in spacetime. According to GR, there is only the sensually and metrologically non-experienceable spacetime.

Despite the impossibility of being able to plausibly depict diverse mathematically generated theoretical objects and interaction scenarios in a generally understandable way, it is didactically widespread to convey knowledge with "simplified" (unfortunately false) intuitions. An example is the inflated balloon with markings (which are supposed to represent the planets, stars, galaxies) that is supposed to depict cosmic expansion after the Big Bang. This creates in the observer a *feeling* of intuitive understanding. Only this balloon analogy is **fundamentally false**. Because...

i) Within the framework of GR, not a sensually perceptible space or, in simplified illustrative form, a balloon surface expands, but the mathematical construct of spacetime.

ii) There is no unique origin; rather, spacetime expands everywhere.

... it is indeed spacetime itself that expands, **but** the galaxies are co-moved. Established postulate: Gravitationally bound objects such as galaxies or galaxy clusters do not expand. Here the question arises how, where, why, and when the boundary transition from "something" not spacetime-expanding to something spacetime-expanding takes place. This question is not answered by the proponents of the ΛCDM model. With respect to the balloon analogy, contrary to this postulate, the markings on the balloon surface also expand for the observer.

iii) According to current ideas, contrary to the radius increase of the balloon concept, the expansion is accelerated. In the "balloon understanding", the radius expands uniformly or slows down (up to a material-dependent maximum before the balloon bursts).

iv) Also concealed are the (superluminal, temporally strongly limited) postulated, **theory-necessary** *inflation phase* as well as the assumptions and effects of the postulated dark matter and dark energy, which do not occur in the balloon model.

26.2.4 What Is It About at a Higher Level? An Introductory "Formula-Free" Inventory

It is anything but trivial to regard space and time as physical "objects". Space and time are primarily "ordering patterns of the understanding". In order to "obtain" physics from these ordering patterns, phenomenological considerations and explanations are required.

Popular scientific accounts of General Relativity (GR) and, in a larger framework, of the Standard Model of cosmology (Lambda-CDM model) are all inadmissible interpretations, since the complexity necessary for understanding is not taken into account. That is as if someone presents Chinese characters to an audience not competent in that script for viewing and discussion. Then anything and nothing can be interpreted into it, since no one has the prerequisites for decoding and no one has the prerequisites for the script design. Put succinctly: In GR, even typographical errors lead to new solutions, and that (already) applies to people who professionally practice differential geometry.

The Standard Model of cosmology is based on a mathematical-theoretical construct with a large number of free parameters. With these continually re-corrected free parameters, a complex interpretive arbitrariness arises that supports any theory that is desired.

26.2.5 Some Words on a Main Problem of GR and on Theoretical Approaches in General: The "Thermodynamic Incompatibility"

[1]... the fata morgana: the connection between information and entropy. Two words that one – according to John von Neumann – both does not understand are linked by a logarithm and a dimension-giving constant. What is even more astonishing: Many cosmologists seriously believe that they thereby bring thermodynamics and its entire conceptual apparatus on board. Hence Hawking's "logic": "The problem in Bekenstein's argumentation was that a black hole, if it possessed a finite entropy proportional to the area of its event horizon, would also have to have a finite temperature. From this it would follow that a black hole could be in equilibrium with thermal radiation at some non-zero temperature. But according to classical concepts, no such equilibrium is possible, since the black hole... would absorb without... emitting." [HAWKING, S. W. (1998), p. 103].

[1]... What the international elite of theoretical cosmologists never took note of – because their young people, given the competition, probably never studied Gibbs' main works – was the simple fact that "their entropy" is rather a "Shannon entropy", namely concerning information, but not a thermodynamic entropy. Thus it is not conjugated to an absolute, i.e., thermodynamic temperature. Consequently, there is also no problem with thermal radiation. Of course, anyone is free – like Bekenstein and Hawking – to "derive" formulas for "their entropy and temperature" from the "event horizon", for example. Only with the system theory according to Gibbs, Falk et al. has all of this nothing to do! There, entropy is a "general physical quantity" with independent meaning, the temperature T nothing other than the partial derivative $(\partial E/\partial S)_{A,B,C,...}$ with the "active" "general physical quantities" $A, B, C, \ldots$ of the respective system held constant. Moreover, the main laws of thermodynamics and the equations of motion derived from them are formulated with process quantities that locally have nothing to do with the Einstein geometry of GR. This immediately leads to the problem of the extent to which the Second Law is compatible with GR at all...

[1]... If one absolutely wants an "informal temperature" conjugated to information, one must derive it from "information theory"; certainly it has nothing to do with the Kelvin temperature and thus certainly nothing with *physical* radiation processes...

[1]... The "Big Bang problem" as "primal singularity" occurs theoretically only if Einstein's credo (: irreversibility is an illusion) is true. This conclusion then means, however, that one consistently negates thermodynamics from the outset and restricts oneself to pure Hamiltonian mechanics as the basis of GR, or at least considers only isentropic processes (no entropy change) in order to at least "save" the phenomenon of background radiation...

[1]Source: Non-mechanistic presentation of the physical disciplines as mathematical system theory – Vilmos Balogh

The gravitational constant G is measured with an uncertainty of 4.7×10^{-5} – the "worst" of all natural constant measurements [3].

The Hubble constant fluctuates by percentages depending on the measurement method [4] – and that with stated error bars of under one percent. This yields a significance of the discrepancy of 4–6σ. The ΛCDM model interprets this finding as "tension" that demands "new physics". The methodologically only permissible interpretation, however, is: A model that produces two such precise but incompatible measurements has falsified itself. The significance does not testify to the accuracy of the standard model of cosmology, but to the depth of its self-contradiction.

Marginal notes: The background radiation was indeed predicted by the (inflationary) Big Bang theory, but little known is that the first predictions were around 50 K and only after the measured values became known in 1965 was the theory "adjusted". Other scientists who, alternatively to the Big Bang thesis, tried to apply the theory of blackbody radiation to space calculated values between 0.75 K (Nernst 1938) and 6 K (Guillaume 1896). If "someone" now, result-oriented within the framework of the ΛCDM model, "calculates" "his" "best fit" with a 10 % smaller value, the question of the predictive capability of the used theory becomes superfluous.

Against this argumentative background, a claim of 10^{-22} is not only implausible, but methodologically absurd.

26.2.6 The Missing Calibratability

A measurement of the order of magnitude 10^{-22} requires perfect knowledge and control of all noise sources:

- thermal noise sources,
- seismic vibrations,
- electromagnetic disturbances,
- quantum mechanical fluctuations.

This control is already extremely difficult in a laboratory experiment – in a large-scale technical facility like LIGO it is impossible. The claim that one can distinguish a signal of 10^{-22} from the noise is a claim *without methodological basis.*

26.2.7 The Missing Repeatability

A scientific experiment must be repeatable. LIGO events are unique and cannot be reproduced. The only "confirmation" comes from other detectors (Virgo), which work with the same methods and the same model assumptions – there is no independent control [5].

26.2.8 Consequence

The LIGO "measurement" is not a measurement in the scientific sense. It is a theory-laden data construction without methodological basis.

26.3 Level 2: The Theoretical Inconsistency of the Gravitational Wave Solutions

26.3.1 GR Has No General Analytical Solutions

Einstein's field equations are a system of nonlinear, coupled differential equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{289}$$

They possess no general analytical solutions [6]. In order to obtain computable solutions at all, idealizations and approximations are required.

26.3.2 The Approximations Violate the Covariance Principle

The covariance principle is a central axiom of GR: The form of the laws of nature should not depend on the choice of coordinate system. The gravitational wave solutions are obtained through *linearization* of the field equations – an approximation for weak fields [7]. This linearization is *not covariant*: It presupposes a certain coordinate choice (a decomposition of space and time) that violates the axiom. What stems from a covariance-violating approximation can claim **no real-physically founded meaning**.

26.3.3 The Embedding in the Newtonian Space Container

The linearized solutions are embedded for the LIGO interpretation in a Newtonian, flat space – they are decomposed into space and time, although GR postulates that space and time are not separable [8]. The theory that ultimately comes to application is *no longer general-relativistic* in the sense of the original axiomatics.

26.3.4 Consequence

Gravitational waves are **artifacts of non-covariant, selective approximations**.

They are mathematical constructs, **not** physical objects.

26.3.5 Mathematics Then and Now

If Euclid (... probably lived in the 3rd century BC) still sought plausible intuition for mathematical foundations and thus established an interdisciplinary connection that could be judged as right or wrong, then in modern mathematics the question of right or wrong does not arise. Euclid's definitions are explicit; they refer to extra-mathematical objects of "pure intuition" such as points, lines, and surfaces. "A point is that which has no breadth. A line is breadthless length. A surface is that which has only length and breadth." When David Hilbert (1862–1943) axiomatized geometry again in the 20th century, he used exclusively implicit definitions. The objects of geometry were still called "points" and "lines", but they were merely elements of sets not further explicated. Hilbert is said to have said that one could at any time speak of tables and chairs instead of points and lines without disturbing the purely logical relationship between these objects.

But to what extent axiomatically founded abstractions couple to real-physical objects is on a completely different page. Mathematics creates no new insights, even if theoretical physicists within the framework of the standard models of cosmology and particle physics like to believe that.

"No church holds its believers as tightly on the leash as 'modern science' holds its congregation." noted *Christian Morgenstern* (1871–1914) more than 100 years ago.

26.4 Level 3: The Epistemological Impossibility of a "Spacetime Measurement"

26.4.1 Spacetime Is Not Measurable

GR postulates spacetime as a four-dimensional continuum in which space and time are inseparably connected [9]. Every physical measurement, however, is *space-time decoupled*: We measure a time difference in a spatial arrangement. A measurement of spacetime itself would presuppose that we can observe it "from outside" – a logical contradiction.

26.4.2 What LIGO Actually Measures

LIGO measures **phase differences of laser light** in an interferometer – nothing else [2]. The interpretation of these phase differences as "spacetime curvature" is a metaphysical act, not a physical one. The cause of the phase differences could be *anything* (thermal effects, electromagnetic disturbances, ununderstood noise sources) – there is no methodological basis to attribute them specifically to gravitational waves.

26.4.3 Consequence

The claim to have “measured” gravitational waves is **epistemologically impossible**. A spacetime measurement is a contradiction in itself.

26.5 Level 4: The Historical and Sociological Dimension

26.5.1 Einstein’s Own Doubts

Einstein himself was unsure from 1936 to 1938 whether gravitational waves exist – and at times came to the conclusion that they *do not exist* [10]. If the creator of the theory doubted the existence of his own wave solutions, today’s certainty of the LIGO community is *not covered by the theory.*

26.5.2 The Immunization Strategy of the ΛCDM Model

The ΛCDM model has been immunized against falsification through the continual addition of new, unobservable parameters (inflation, dark matter, dark energy) [11]. Gravitational waves and black holes are *necessary constructs* of this model – their supposed detection distracts from the actual model inconsistencies. The entire model would collapse if these constructs were recognized as what they are: artifacts of non-covariant approximations.

26.5.3 The Suppression of Criticism

Critics like Halton Arp [12] or Dirk Freyling, Elementary Body Theory (EBT) [14], are systematically excluded from scientific discourse (publication refusal, withdrawal of resources, damage to reputation). The scientific community has invested itself in a paradigm that can no longer be questioned without bringing down the entire edifice.

26.5.4 Consequence

The LIGO “detections” are not scientific proof – they are a **sociological phenomenon** maintained by the power of established paradigms.

26.6 The Balance

- The EBT derivation of the perihelion shift is **more transparent** than the GR derivation, because it explicitly names its settings.
- EBT and GR use **the same mathematics** (perturbation calculation, Newtonian relation $p = a(1 - e^2)$).
- The difference lies in the **ontology**: GR interprets the factor 3 as spacetime curvature, EBT as transverse motion coupling in the space-energy field.
- EBT has **only one scalar field equation** and therefore cannot determine g_{tt} and g_{rr} independently. It needs Theorem 3 (scale argument) to fix g_{rr}.
- Theorem 3 is a **theorem**, not a postulate. It follows compellingly from Axiom 1 + Axiom 2 + mass–radius coupling, applied to the scale itself. The universal scaling of local energy is **part of the definition of Axiom 1**, not an additional setting.
- The state equation $\nabla^2\Phi = +4\pi\gamma_G\rho\Phi/c^2$ is an **independent postulate** of EBT. It is not derived from the Freyling intervention.
- The GR derivation is a **back-calculation** that reproduces the known result through idealizations and Newton imports. The EBT derivation is a **deduction** from phenomenologically founded axioms and geometric theorems.

27 Appendix

27.1 Cosmological GTR and ΛCDM

Complete derivation from the Einstein equation to Friedmann, $H(z)$, and distances

27.1.1 Einstein equation with cosmological constant

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

27.1.2 Homogeneity and isotropy as cosmological postulate

The spatial hypersurfaces are assumed to be homogeneous and isotropic. Thereby the general metric is reduced to the FLRW form:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right]. \tag{2}$$

The unknown cosmic dynamics is contained in the scale factor $a(t)$. k is the constant spatial curvature parameter.

27.1.3 Metric components

$$g_{\mu\nu} = \mathrm{diag}\left(-c^2, \frac{a^2}{1 - kr^2}, a^2 r^2, a^2 r^2 \sin^2\theta \right). \tag{3}$$

The inverse metric is

$$g^{\mu\nu} = \mathrm{diag}\left(-\frac{1}{c^2}, \frac{1 - kr^2}{a^2}, \frac{1}{a^2 r^2}, \frac{1}{a^2 r^2 \sin^2\theta} \right). \tag{4}$$

Define

$$H = \frac{\dot{a}}{a}. \tag{5}$$

27.1.4 Selected Christoffel symbols

From

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right) \tag{6}$$

it follows among others

$$\Gamma^0_{rr} = \frac{a\dot{a}}{c^2(1 - kr^2)}, \tag{7}$$

$$\Gamma^0_{\theta\theta} = \frac{a\dot{a}}{c^2} r^2, \tag{8}$$

$$\Gamma^0_{\phi\phi} = \frac{a\dot{a}}{c^2} r^2 \sin^2\theta, \tag{9}$$

and

$$\Gamma^r_{0r} = \Gamma^\theta_{0\theta} = \Gamma^\phi_{0\phi} = H. \tag{10}$$

27.1.5 Ricci tensor

After substituting the Christoffel symbols into

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\mu\nu} \Gamma^\sigma_{\lambda\sigma} - \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} \tag{11}$$

one obtains

$$R_{00} = -3\frac{\ddot{a}}{a}. \tag{12}$$

The spatial components have the common structure

$$R_{ij} = \frac{a\ddot{a} + 2\dot{a}^2 + 2kc^2}{c^2}\gamma_{ij}, \tag{13}$$

where γ_{ij} denotes the time-independent spatial unit metric.

The Ricci scalar is

$$R = \frac{6}{c^2}\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2}\right]. \tag{14}$$

27.1.6 Einstein tensor

For the 00-component one obtains

$$G_{00} = R_{00} - \frac{1}{2}g_{00}R. \tag{15}$$

Since $g_{00} = -c^2$:

$$G_{00} = -3\frac{\ddot{a}}{a} + \frac{c^2}{2}\frac{6}{c^2}\left[\frac{\ddot{a}}{a} + H^2 + \frac{kc^2}{a^2}\right] \tag{16}$$

$$= 3\left(H^2 + \frac{kc^2}{a^2}\right). \tag{17}$$

Thus

$$G_{00} = 3\left(H^2 + \frac{kc^2}{a^2}\right). \tag{18}$$

27.1.7 Matter tensor as ideal fluid

Homogeneity and isotropy enforce for matter the perfect-fluid form

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right)u_\mu u_\nu + pg_{\mu\nu}. \tag{19}$$

In the comoving system:

$$u^\mu = (1, 0, 0, 0) \tag{20}$$

using the cosmic time convention. From this follows

$$T_{00} = \rho c^2. \tag{21}$$

27.1.8 First Friedmann equation

The 00-Einstein equation reads

$$G_{00} + \Lambda g_{00} = \frac{8\pi G}{c^4}T_{00}. \tag{22}$$

Substitution:

$$3\left(H^2 + \frac{kc^2}{a^2}\right) - \Lambda c^2 = \frac{8\pi G}{c^2}\rho. \tag{23}$$

With the convention used here for ρ as mass density it follows that

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}. \tag{24}$$

27.1.9 Second Friedmann equation

From a spatial diagonal component one obtains after substituting $T_{ij} = pg_{ij}$:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}. \tag{25}$$

27.1.10 Continuity equation

The contracted Bianchi identity enforces

$$\nabla_\mu G^{\mu\nu} = 0. \tag{26}$$

Thus from the Einstein equation follows

$$\nabla_\mu T^{\mu\nu} = 0. \tag{27}$$

For the FLRW fluid, the $\nu = 0$-component gives

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0. \tag{28}$$

27.1.11 Equation of state and density scaling

Set for a component

$$p = w\rho c^2. \tag{29}$$

Then

$$\dot{\rho} + 3H\rho(1 + w) = 0. \tag{30}$$

With

$$H = \frac{\dot{a}}{a} \tag{31}$$

becomes

$$\frac{\dot{\rho}}{\rho} = -3(1 + w)\frac{\dot{a}}{a}. \tag{32}$$

Integration:

$$\ln \rho = -3(1 + w) \ln a + \text{const}. \tag{33}$$

Thus

$$\rho(a) = \rho_0 a^{-3(1+w)}. \tag{34}$$

Special cases:

$$w = 0 : \qquad \rho_m = \rho_{m0} a^{-3}, \tag{35}$$

$$w = \frac{1}{3} : \qquad \rho_r = \rho_{r0} a^{-4}, \tag{36}$$

$$w = -1 : \qquad \rho_\Lambda = \text{const}. \tag{37}$$

27.1.12 Density parameters

Define today

$$\rho_c = \frac{3H_0^2}{8\pi G}. \tag{38}$$

Then

$$\Omega_m = \frac{\rho_{m0}}{\rho_c}, \qquad \Omega_r = \frac{\rho_{r0}}{\rho_c}, \tag{39}$$

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}, \qquad \Omega_k = -\frac{kc^2}{H_0^2}. \tag{40}$$

With $a_0 = 1$ and

$$1 + z = \frac{1}{a} \tag{41}$$

it follows that

$$H(z)^2 = H_0^2 \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]. \tag{42}$$

27.1.13 Light propagation and distance

For radially traveling light:

$$ds^2 = 0. \tag{43}$$

Thus

$$c^2 dt^2 = a(t)^2 \frac{dr^2}{1 - kr^2}. \tag{44}$$

Define the radial comoving coordinate χ by

$$d\chi = \frac{dr}{\sqrt{1 - kr^2}}. \tag{45}$$

Then

$$d\chi = \frac{c\,dt}{a(t)}. \tag{46}$$

With

$$1 + z = \frac{1}{a}, \qquad \frac{dz}{dt} = -(1+z)H(z) \tag{47}$$

it follows that

$$dt = -\frac{dz}{(1+z)H(z)}. \tag{48}$$

Since $a = 1/(1+z)$:

$$\frac{dt}{a} = -\frac{dz}{H(z)}. \tag{49}$$

Thus

$$\chi(z) = c \int_0^z \frac{dz'}{H(z')}. \tag{50}$$

For $k = 0$:

$$D_M = \chi. \tag{51}$$

Angular diameter distance:

$$D_A = \frac{\chi}{1+z}. \tag{52}$$

Luminosity distance:

$$D_L = (1+z)\chi. \tag{53}$$

Distance modulus:

$$\mu = 5 \log_{10}\left(\frac{D_L}{10\,\mathrm{pc}}\right). \tag{54}$$

27.1.14 Where does ΛCDM begin to go beyond the pure FLRW reduction?

The Friedmann equations do not determine $a(t)$ without specification of the matter components and their equations of state. ΛCDM additionally specifies a concrete composition:

$$\text{radiation} + \text{baryonic matter} + \text{cold dark matter} + \Lambda. \tag{55}$$

For real structure formation, the homogeneous solution is perturbed:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{56}$$

$$\rho = \bar{\rho} + \delta\rho. \tag{57}$$

Linear cosmology retains only terms of first order in the perturbations.

27.1.15 Methodical chain

$G_{\mu\nu}+\Lambda g_{\mu\nu} \to$ homogeneity + isotropy $\to$ FLRW $\to T_{\mu\nu}^{\text{Fluid}} \to$ Friedmann I + II $\to \nabla_\mu T^{\mu\nu} = 0 \to p = w\rho c^2 \to \rho(a) \to H($

27.2 Complete Derivation of the Perihelion Precession of Mercury in the General Theory of Relativity

27.2.1 Goal and Methodical Separation

The derivation is carried out completely from the covariant origin to the known approximation formula for the perihelion precession. Four levels are kept apart:

1. the generally covariant Einstein field equation,
2. the physical-geometric postulates for a static, spherically symmetric source,
3. the exact Schwarzschild solution and the exact geodesic equation,
4. the perturbation calculation for Mercury's orbit, which begins only afterwards.

The well-known formula

$$\Delta\phi \simeq \frac{6\pi GM}{c^2 a(1-e^2)} \tag{1}$$

is therefore not the immediate output of the Einstein field equation but the end of a chain consisting of symmetry reduction, exact integration, geodesic dynamics, and subsequent approximation solution.

27.2.2 The Covariant Origin

The Einstein field equation without cosmological constant reads

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

$$R = g^{\alpha\beta} R_{\alpha\beta} \tag{3}$$

The Ricci tensor is

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\mu\nu}\Gamma^\sigma_{\lambda\sigma} - \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} \tag{4}$$

and the Christoffel symbols are

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right) \tag{5}$$

For the exterior region of the Sun, one sets

$$T_{\mu\nu} = 0 \tag{6}$$

$$G_{\mu\nu} = 0 \tag{7}$$

By contraction one obtains in vacuum

$$R = 0 \tag{8}$$

$$R_{\mu\nu} = 0 \tag{9}$$

This is the exact starting equation for the Schwarzschild solution.

27.2.3 The General Tensor and the Symmetry Reduction

A symmetric metric tensor possesses in four dimensions initially ten independent components

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix} \tag{10}$$

Correspondingly, the Einstein tensor is initially

$$G_{\mu\nu} = \begin{pmatrix} G_{00} & G_{01} & G_{02} & G_{03} \\ G_{01} & G_{11} & G_{12} & G_{13} \\ G_{02} & G_{12} & G_{22} & G_{23} \\ G_{03} & G_{13} & G_{23} & G_{33} \end{pmatrix} \tag{11}$$

For the classical Mercury calculation, however, from the outset a static, spherically symmetric metric is assumed:

$$ds^2 = -e^{2\nu(r)}c^2dt^2 + e^{2\lambda(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \tag{12}$$

Thereby the following postulates are already made:

- Staticity:

$$\partial_t g_{\mu\nu} = 0.$$

- Spherical symmetry: the unknown functions depend only on r.
- No rotation of the source.
- No multipole structure of the source.
- No external bodies.
- No time-dependent fields.
- Areal radius r:

$$g_{\theta\theta} = r^2$$

$$g_{\phi\phi} = r^2$$

$\sin^2\theta$.

The metric matrix thus reads

$$g_{\mu\nu} = \begin{pmatrix} -c^2e^{2\nu(r)} & 0 & 0 & 0 \\ 0 & e^{2\lambda(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \tag{13}$$

and the inverse metric tensor

$$g^{\mu\nu} = \begin{pmatrix} -e^{-2\nu(r)}/c^2 & 0 & 0 & 0 \\ 0 & e^{-2\lambda(r)} & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/(r^2\sin^2\theta) \end{pmatrix} \tag{14}$$

27.2.4 Non-vanishing Christoffel Symbols

For the metric (12), the independent non-vanishing Christoffel symbols are:

$$\Gamma^t_{tr} = \Gamma^t_{rt} = \nu' \tag{15}$$

$$\Gamma^r_{tt} = c^2 e^{2(\nu-\lambda)}\nu' \tag{16}$$

$$\Gamma^r_{rr} = \lambda' \tag{17}$$

$$\Gamma^r_{\theta\theta} = -re^{-2\lambda} \tag{18}$$

$$\Gamma^r_{\phi\phi} = -re^{-2\lambda}\sin^2\theta \tag{19}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r} \tag{20}$$

$$\Gamma^\phi_{r\phi} = \Gamma^\phi_{\phi r} = \frac{1}{r} \tag{21}$$

$$\Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta \tag{22}$$

$$\Gamma^\phi_{\theta\phi} = \Gamma^\phi_{\phi\theta} = \cot\theta \tag{23}$$

As four 4 × 4 matrices:

$$\Gamma^t_{\mu\nu} = \begin{pmatrix} 0 & \nu' & 0 & 0 \\ \nu' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{24}$$

$$\Gamma^r_{\mu\nu} = \begin{pmatrix} c^2 e^{2(\nu-\lambda)}\nu' & 0 & 0 & 0 \\ 0 & \lambda' & 0 & 0 \\ 0 & 0 & -re^{-2\lambda} & 0 \\ 0 & 0 & 0 & -re^{-2\lambda}\sin^2\theta \end{pmatrix} \tag{25}$$

$$\Gamma^{\theta}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1/r & 0 \\ 0 & 1/r & 0 & 0 \\ 0 & 0 & 0 & -\sin\theta\cos\theta \end{pmatrix} \tag{26}$$

$$\Gamma^{\phi}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/r \\ 0 & 0 & 0 & \cot\theta \\ 0 & 1/r & \cot\theta & 0 \end{pmatrix} \tag{27}$$

Up to this point, no weak-field approximation has been made.

27.2.5 Ricci Tensor and Einstein Tensor

From the Christoffel symbols follows

$$R_{tt} = c^2 e^{2(\nu-\lambda)} \left[\nu'' + \nu'^2 - \nu'\lambda' + \frac{2\nu'}{r}\right] \tag{28}$$

$$R_{rr} = -\nu'' - \nu'^2 + \nu'\lambda' + \frac{2\lambda'}{r} \tag{29}$$

$$R_{\theta\theta} = 1 - e^{-2\lambda}\left[1 + r(\nu' - \lambda')\right] \tag{30}$$

$$R_{\phi\phi} = \sin^2\theta\, R_{\theta\theta} \tag{31}$$

Thus

$$R_{\mu\nu} = \begin{pmatrix} R_{tt} & 0 & 0 & 0 \\ 0 & R_{rr} & 0 & 0 \\ 0 & 0 & R_{\theta\theta} & 0 \\ 0 & 0 & 0 & \sin^2\theta\, R_{\theta\theta} \end{pmatrix} \tag{32}$$

For the mixed components of the Einstein tensor one obtains

$$G^t{}_t = -\frac{1}{r^2} + e^{-2\lambda}\left(\frac{1}{r^2} - \frac{2\lambda'}{r}\right) \tag{33}$$

$$G^r{}_r = -\frac{1}{r^2} + e^{-2\lambda}\left(\frac{1}{r^2} + \frac{2\nu'}{r}\right) \tag{34}$$

$$G^\theta{}_\theta = G^\phi{}_\phi = e^{-2\lambda}\left[\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu' - \lambda'}{r}\right] \tag{35}$$

$$G^\mu{}_\nu = 0 \tag{36}$$

The angular equation is, due to the contracted Bianchi identity

$$\nabla_\mu G^\mu{}_\nu = 0 \tag{37}$$

not independent of the two radial equations.

27.2.6 Exact Integration to the Schwarzschild Solution

$$-\frac{1}{r^2} + e^{-2\lambda}\left(\frac{1}{r^2} - \frac{2\lambda'}{r}\right) = 0 \tag{38}$$

$$f(r) = e^{-2\lambda(r)} \tag{39}$$

$$f' = -2\lambda' f \tag{40}$$

The equation becomes

$$-\frac{1}{r^2} + \frac{f}{r^2} + \frac{f'}{r} = 0 \tag{41}$$

$$-1 + f + rf' = 0 \tag{42}$$

$$\frac{d}{dr}(rf) = f + rf' \tag{43}$$

$$\frac{d}{dr}(rf) = 1 \tag{44}$$

$$rf = r + C_1 \tag{45}$$

$$f(r) = 1 + \frac{C_1}{r} \tag{46}$$

$$e^{-2\lambda} = 1 - \frac{C}{r} \tag{47}$$

Now (34) is used:

$$-\frac{1}{r^2} + e^{-2\lambda}\left(\frac{1}{r^2} + \frac{2\nu'}{r}\right) = 0 \tag{48}$$

$$-1 + f + 2rf\nu' = 0 \tag{49}$$

$$\nu' = \frac{C}{2r(r-C)} \tag{50}$$

From (47) simultaneously follows

$$\lambda' = -\frac{C}{2r(r-C)} \tag{51}$$

$$\nu' = -\lambda' \tag{52}$$

$$\nu = -\lambda + K \tag{53}$$

$$e^{2\nu} = e^{2K}e^{-2\lambda} \tag{54}$$

Asymptotic flatness demands

$$r \to \infty : \quad g_{tt} \to -c^2 \tag{55}$$

$$e^{2K} = 1 \tag{56}$$

$$e^{2\nu} = 1 - \frac{C}{r} \tag{57}$$

Up to here, C is merely an integration constant. For identification with the source mass, the weak static limiting case is invoked:

$$g_{tt} \simeq -c^2\left(1 + \frac{2\Phi_N}{c^2}\right), \qquad \Phi_N = -\frac{GM}{r} \tag{58}$$

$$g_{tt} \simeq -c^2 \left(1 - \frac{2GM}{c^2 r}\right) \tag{59}$$

$$g_{tt} = -c^2 \left(1 - \frac{C}{r}\right) \tag{60}$$

$$C = \frac{2GM}{c^2} \equiv r_s \tag{61}$$

Thus follows the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \tag{62}$$

$$r_s = \frac{2GM}{c^2} \tag{63}$$

Within the stated symmetry and vacuum assumptions, (62) is an exact solution. A weak-field development of the metric was not used to solve the vacuum equations; the Newtonian limiting case serves here for the physical identification of the integration constant C.

27.2.7 Exact Geodesic Equation for Mercury

$$f(r) = 1 - \frac{r_s}{r} \tag{64}$$

$$ds^2 = -f c^2 dt^2 + f^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \tag{65}$$

For a massive test body, exactly:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0 \tag{66}$$

$$\dot{x} = \frac{dx}{d\tau} \tag{67}$$

The four components are:

$$\ddot{t} + \frac{f'}{f} \dot{t} \, \dot{r} = 0 \tag{68}$$

This is exactly integrable:

$$\frac{d}{d\tau}(f\dot{t}) = 0 \tag{69}$$

$$E = fc^2\dot{t} = \text{const.} \tag{70}$$

$$\ddot{r} + \frac{c^2}{2}ff'\dot{t}^2 - \frac{f'}{2f}\dot{r}^2 - rf\dot{\theta}^2 - rf\sin^2\theta\,\dot{\phi}^2 = 0 \tag{71}$$

$$\ddot{\theta} + \frac{2}{r}\dot{r}\,\dot{\theta} - \sin\theta\cos\theta\,\dot{\phi}^2 = 0 \tag{72}$$

Due to spherical symmetry, the orbital plane can be chosen as the equatorial plane:

$$\theta = \frac{\pi}{2}, \qquad \dot{\theta} = 0 \tag{73}$$

$$\ddot{\phi} + \frac{2}{r}\dot{r}\,\dot{\phi} = 0 \tag{74}$$

This is equivalent to

$$\frac{d}{d\tau}(r^2\dot{\phi}) = 0 \tag{75}$$

$$h = r^2\dot{\phi} = \text{const.} \tag{76}$$

27.2.8 Exact Radial Energy Equation

For a timelike geodesic, the normalization holds

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -c^2 \tag{77}$$

$$-fc^2\dot{t}^2 + f^{-1}\dot{r}^2 + r^2\dot{\phi}^2 = -c^2 \tag{78}$$

$$\dot{t} = \frac{E}{fc^2}, \qquad \dot{\phi} = \frac{h}{r^2} \tag{79}$$

$$-\frac{E^2}{fc^2} + \frac{\dot{r}^2}{f} + \frac{h^2}{r^2} = -c^2 \tag{80}$$

$$-\frac{E^2}{c^2} + \dot{r}^2 + f\frac{h^2}{r^2} = -fc^2 \tag{81}$$

$$\dot{r}^2 = \frac{E^2}{c^2} - f\left(c^2 + \frac{h^2}{r^2}\right) \tag{82}$$

$$f = 1 - \frac{2GM}{c^2 r} \tag{83}$$

after expansion yields

$$\dot{r}^2 = \frac{E^2}{c^2} - c^2 - \frac{h^2}{r^2} + \frac{2GM}{r} + \frac{2GMh^2}{c^2 r^3} \tag{84}$$

The $1/r^3$-term here is not yet a result of a weak-field perturbation expansion but follows algebraically from the Schwarzschild geodesic dynamics.

27.2.9 Exact Binet Equation

$$u(\phi) = \frac{1}{r} \tag{85}$$

$$\dot{\phi} = hu^2 \tag{86}$$

$$\frac{dr}{d\phi} = -\frac{1}{u^2}\frac{du}{d\phi} \tag{87}$$

$$\dot{r} = \frac{dr}{d\phi}\dot{\phi} = -hu' \tag{88}$$

$$u' = \frac{du}{d\phi} \tag{89}$$

$$h^2(u')^2 = \frac{E^2}{c^2} - c^2 - h^2u^2 + 2GMu + \frac{2GMh^2}{c^2}u^3 \tag{90}$$

$$2h^2u'u'' = -2h^2uu' + 2GMu' + \frac{6GMh^2}{c^2}u^2u' \tag{91}$$

For $u' \neq 0$, one may divide by $2h^2u'$; the resulting differential equation holds by continuous extension also at the radial turning points:

$$u'' = -u + \frac{GM}{h^2} + \frac{3GM}{c^2}u^2 \tag{92}$$

$$u'' + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2 \tag{93}$$

Within the Schwarzschild test-body description, (93) is not yet a linearized weak-field equation.

27.2.10 Beginning of the Approximation: Perturbation Calculation

The approximation now begins with the solution of the nonlinear equation (93).

$$p = \frac{h^2}{GM} \tag{94}$$

The equation reads

$$u'' + u = \frac{1}{p} + \frac{3GM}{c^2} u^2 \tag{95}$$

As zeroth-order solution, the Kepler form is used:

$$u_0(\phi) = \frac{1}{p}(1 + e\cos\phi) \tag{96}$$

$$u_0'' + u_0 = \frac{1}{p} \tag{97}$$

Now one sets

$$u = u_0 + \delta u \tag{98}$$

Substitution into the exact Binet equation yields

$$\delta u'' + \delta u = \frac{3GM}{c^2}\left(u_0^2 + 2u_0\delta u + (\delta u)^2\right) \tag{99}$$

For the small parameter

$$\epsilon = \frac{GM}{c^2 p} \ll 1 \tag{100}$$

δu is treated as a first-order quantity. Terms of higher order are discarded:

$$\delta u'' + \delta u \simeq \frac{3GM}{c^2} u_0^2 \tag{101}$$

$$u_0^2 = \frac{1}{p^2}\left(1 + 2e\cos\phi + e^2\cos^2\phi\right) \tag{102}$$

$$\delta u'' + \delta u \simeq \frac{3GM}{c^2p^2}\left(1 + 2e\cos\phi + e^2\cos^2\phi\right) \tag{103}$$

For the secular perihelion precession, the resonant component

$$\frac{6GMe}{c^2p^2}\cos\phi \tag{104}$$

is decisive.

$$y'' + y = A\cos\phi \tag{105}$$

a resonant particular solution is

$$y_{\mathrm{res}} = \frac{A}{2}\,\phi\sin\phi \tag{106}$$

$$\delta u_{\mathrm{res}} = \frac{3GMe}{c^2p^2}\phi\sin\phi \tag{107}$$

Thus the orbit in first order contains

$$u \simeq \frac{1}{p}\left[1 + e\cos\phi + e\frac{3GM}{c^2p}\phi\sin\phi + \dots\right] \tag{108}$$

$$\cos[(1-\delta)\phi] \simeq \cos\phi + \delta\phi\sin\phi \tag{109}$$

$$\delta = \frac{3GM}{c^2p} \tag{110}$$

A radial period is reached when

$$(1-\delta)\phi = 2\pi \tag{111}$$

$$\phi = \frac{2\pi}{1-\delta} \simeq 2\pi(1+\delta) \tag{112}$$

The additional angular displacement per revolution is thus

$$\Delta\phi = \phi - 2\pi \simeq 2\pi\delta \tag{113}$$

$$\Delta\phi \simeq \frac{6\pi GM}{c^2 p} \tag{114}$$

With the relation used in first order

$$p = a(1 - e^2) \tag{115}$$

$$\Delta\phi \simeq \frac{6\pi GM}{c^2 a(1 - e^2)} \tag{116}$$

per revolution.

27.2.11 Numerical Evaluation for Mercury

For the Sun and Mercury, we use as an example

$$G = 6.67430 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2} \tag{117}$$

$$M_\odot \approx 1.98847 \times 10^{30}\,\mathrm{kg} \tag{118}$$

$$c = 2.99792458 \times 10^{8}\,\mathrm{m/s} \tag{119}$$

$$a \approx 5.790905 \times 10^{10}\,\mathrm{m} \tag{120}$$

$$e \approx 0.205630 \tag{121}$$

Then from (116):

$$\Delta\phi \approx 5.02 \times 10^{-7}\,\mathrm{rad/revolution} \tag{122}$$

$$1\,\mathrm{rad} = 206264.806\ldots'' \tag{123}$$

$$\Delta\phi \approx 0.1035''/\mathrm{revolution} \tag{124}$$

The orbital period of Mercury is approximately

$$87.9691 \,\text{days} \tag{125}$$

A Julian century contains

$$36525 \,\text{days} \tag{126}$$

$$N = \frac{36525}{87.9691} \approx 415.20 \tag{127}$$

$$\Delta\phi_{\text{Jh}} \approx 42.98'' \approx 43.0'' \tag{128}$$

per century.

27.2.12 Explicit List of Postulates and Approximations

Before solving the field equation

1. static source,
2. exact spherical symmetry,
3. no rotation,
4. no multipole structure,
5. no external bodies,
6. exterior region:
$$T_{\mu\nu} = 0,$$
7. choice of areal radius and Schwarzschild coordinates.

In the physical identification of the vacuum solution

The vacuum equation initially provides only the integration constant C. The identification

$$C = \frac{2GM}{c^2} \tag{129}$$

is made by matching to the weak Newtonian limiting case or, equivalently, by the physical specification of the asymptotic mass parameter.

In the test-body motion

1. Mercury is treated as a test body.
2. Its back-reaction on the metric is neglected.

3. Other planets are not considered in the Schwarzschild two-body calculation.

4. The orbital plane is chosen, due to spherical symmetry, as

$$\theta = \frac{\pi}{2}.$$

Only in the solution of the Binet equation

The exact Schwarzschild Binet equation reads

$$u'' + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2. \tag{130}$$

The approximation begins with

$$u = u_0 + \delta u \tag{131}$$

and the truncation after first order in

$$\epsilon = \frac{GM}{c^2 p}. \tag{132}$$

Additionally, the expansions

$$\cos[(1-\delta)\phi] \simeq \cos\phi + \delta\phi\sin\phi \tag{133}$$

and

$$\frac{1}{1-\delta} \simeq 1 + \delta \tag{134}$$

are used.

27.2.13 The Entire Derivation as a Chain

$$\begin{aligned} G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} &\Longrightarrow \text{staticity + spherical symmetry + areal radius} \Longrightarrow T_{\mu\nu} = 0 \Longrightarrow R_{\mu\nu} = 0 \\ &\Longrightarrow G^t{}_t = 0, \quad G^r{}_r = 0 \Longrightarrow e^{-2\lambda} = 1 - \frac{C}{r}, \quad e^{2\nu} = 1 - \frac{C}{r} \\ &\Longrightarrow C = \frac{2GM}{c^2} \Longrightarrow \text{Schwarzschild metric} \Longrightarrow \text{exact geodesic equation} \\ &\Longrightarrow E = fc^2\dot{t}, \quad h = r^2\dot{\phi} \Longrightarrow \dot{r}^2 = \frac{E^2}{c^2} - f\left(c^2 + \frac{h^2}{r^2}\right) \\ &\Longrightarrow u = \frac{1}{r} \Longrightarrow u'' + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2 \\ &\Longrightarrow u = u_0 + \delta u \Longrightarrow \text{first-order perturbation expansion} \\ &\Longrightarrow \Delta\phi \simeq \frac{6\pi GM}{c^2 a(1-e^2)} \Longrightarrow \Delta\phi_{\text{Mercury}} \approx 43.0''/\text{century}. \end{aligned} \tag{135}$$

27.2.14 Interim Conclusion

The mathematical autopsy separates three distinct facts:

1. The Schwarzschild solution is, within the static, spherically symmetric vacuum postulate, an exact solution of the nonlinear Einstein equations.

2. The Binet equation
$$u'' + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2$$
follows within the Schwarzschild test-body description likewise without weak-field perturbation expansion.

3. The well-known closed 6π-formula arises only through the perturbation calculation around the Kepler form and the truncation after first order.

The number of approximately $43''$ per century is therefore not immediately the Einstein tensor equation itself but the end result of a specialized and subsequently perturbatively solved model chain.

Separate from this is the observational side: The historically stated anomalous Mercury residual value is not a single quantity read directly from a measuring instrument but arises after modeling and subtraction of further contributions to the observed Mercury motion.

27.3 The Perihelion Precession of Mercury – Historical and Modern Autopsy of the Famous $43''$ Test

Extension to the document

The Derivation of the Perihelion Precession: The Honest Autopsy

From Observation via Newcomb's Adjustment and Einstein's Input Quantities to Radar, MESSENGER and Modern Ephemerides

Working Document 2026

27.3.1 Why This Extension Is Necessary

The popular short form of the Mercury test reads approximately: The General Theory of Relativity predicts for Mercury an additional perihelion precession of about $43''$ per century, and precisely this value is observed. This formulation is didactically catchy but renders the actual measurement and calculation path invisible. Neither historically nor today is a number of $43''$ read directly from an instrument. Likewise, the number $42.98''$ on the theoretical side is not a dimensionless output of the Einstein field equation. It arises only after substituting astronomically determined quantities into a specialized and perturbatively evaluated formula. The following presentation therefore consistently separates four levels:

1. What is actually observed?

2. Which astronomical parameters are determined from it?

3. How does the historical residual arise?

4. Which of these parameters subsequently enter into the relativistic calculation?

The goal is not to insinuate anything against historical authors. The reconstruction is merely intended to make visible how, from observation, orbital theory, adjustment, and relativistic calculation, those two numbers arise that in the popular presentation appear almost like two mutually independent measured values.

27.3.2 What Is Usually Communicated – and What Is Historically Documented?

Short form	Historically and measurement-theoretically more precise
"Mercury's perihelion rotates by $43''$."	The entire observed secular motion of the perihelion is much larger. The famous value is an additional residual after subtraction or modeling of other contributions.
"Le Verrier measured $43''$."	Le Verrier found in 1859 initially an unexplained deviation of approximately $38''$ per century. The later, more precise Newcomb evaluation led close to the now famous $43''$ value.
"Einstein predicted $42.98''$."	Einstein obtained in 1915 approximately $43''$ from a formula into which astronomically determined orbital quantities enter. The now precisely cited value $42.98''$ is based on modern constants and orbital elements; Nobili and Will in 1986 drew attention to the long dissemination of a slightly deviating value.
"Experiment: 43 versus theory: 43."	Historically, a residual of a large astronomical adjustment problem is compared with the result of a relativistic formula. Today, tracking data are evaluated in comprehensive numerical ephemeris models.

27.3.3 The Historical Chronology

27.3.3.1 Le Verrier: The Problem Exists Before the GTR Urbain Le Verrier analyzed the motion of Mercury within Newtonian celestial mechanics. After accounting for the then-known planetary perturbations, an additional secular motion of the perihelion remained. The first famous order of magnitude was approximately

$$\Delta\dot{\varpi}_{\rm anom} \sim 38''/\mathrm{Jh}. \tag{1}$$

and not a number already fixed to many decimal places, $43.000''$. What is decisive is the methodical status: Le Verrier did not observe an isolated "relativistic angle." He compared a large number of astronomical positional observations with a theoretically calculated planetary motion. The unexplained residual was parameterized as an additional secular perihelion motion.

27.3.3.2 Newcomb 1895: An Astronomical Problem Becomes a Coupled Constants and Orbital Elements Problem Simon Newcomb's work *The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy* of 1895 is central to the historical reconstruction. Even the structure of the work shows that no single quantity was determined in isolation. Newcomb deals among other things with:

- comparisons of observations and planetary tables,
- corrections of the orbital elements,
- secular variations,
- planetary masses,
- solar parallax,

- precession constants,
- transit observations,
- equations of condition,
- combined solutions and final orbital elements.

Newcomb himself points out in the preface that decisions about one quantity could be changed by later decisions about other quantities. The historical data set and the astronomical constants thus explicitly formed a coupled adjustment system.

27.3.3.3 Newcomb's Final Mercury Value In Newcomb's final secular solution, an additional perihelion advance appears for Mercury of

$$43.37''/\mathrm{Jh.} \tag{2}$$

The frequently reproduced historical decomposition can be schematically represented as

$$575.1''/\mathrm{Jh.} - 531.7''/\mathrm{Jh.} \approx 43.4''/\mathrm{Jh.} \tag{3}$$

where the first number already represents an astronomically reduced secular perihelion motion and the second number represents the calculated Newtonian planetary perturbation. This subtraction is didactically helpful but must not be misunderstood as a subtraction of two directly measured raw values. Both quantities stand at the end of extensive astronomical modeling.

27.3.4 How Was Anything Actually "Measured" Around 1900?

27.3.4.1 The Primary Observations Before radar and spaceflight, primarily optical observations were available:

- meridian observations,
- right ascensions and declinations,
- Mercury transits across the solar disk,
- relative positions with respect to star catalogs,
- long-term series from various observatories.

A telescope thus initially provides angle and time information. It does not directly provide: a, e, $GM_\odot$, $\dot{\varpi}_{\mathrm{anomal}}$. These quantities arise only through astronomical reduction and orbital fitting.

27.3.4.2 The Mathematical Form of an Adjustment Let O_i be the i-th observation and $C_i(q)$ the value calculated on the basis of an orbital system. For a preliminary parameter set q_0, one linearizes:

$$O_i - C_i(\mathbf{q}_0) \simeq \sum_j \frac{\partial C_i}{\partial q_j}\,\delta q_j \tag{4}$$

$$\mathbf{r} = A\,\delta\mathbf{q} \tag{5}$$

$$\mathbf{r} = \begin{pmatrix} O_1 - C_1 \\ O_2 - C_2 \\ \vdots \end{pmatrix} \tag{6}$$

the residual vector and

$$A_{ij} = \frac{\partial C_i}{\partial q_j} \tag{7}$$

the design matrix. The unknown corrections can schematically contain:

$$\delta\mathbf{q} = (\delta a, \delta e, \delta n, \delta\varpi, \delta\dot{\varpi}, \delta i, \delta\dot{\Omega}, \delta M_{\text{Venus}}, \ldots)^T \tag{8}$$

For weighted least squares, one obtains

$$\delta\mathbf{q} = (A^T W A)^{-1} A^T W \mathbf{r} \tag{9}$$

This immediately makes visible: Several orbital quantities are determined jointly from the same observations. They can possess different observational signatures and yet be statistically coupled with each other.

27.3.5 The Parameters of the Einstein Formula: What Do They Mean and How Were They Determined?

27.3.5.1 a: Semi-Major Axis In the well-known modern form

$$\Delta\varpi_{\text{GR}} = \frac{6\pi G M_\odot}{c^2 a(1-e^2)} \tag{10}$$

a is the semi-major axis of the Kepler ellipse associated with the orbit. For Mercury, it is today approximately

$$a \approx 5.79 \times 10^{10}\,\text{m} \approx 0.387\,\text{AU} \tag{11}$$

Geometrically, for an ellipse:

$$a = \frac{r_{\mathrm{peri}} + r_{\mathrm{aph}}}{2} \tag{12}$$

Historically, however, a was not measured directly as the Sun–Mercury distance. Initially, the relative orbit was determined from angular observations and orbital theory. To convert this relative quantity into kilometers or meters, one needed an absolute scale of the solar system.

27.3.5.2 The Astronomical Unit and the Solar Parallax The historical absolute scale was essentially represented by the solar parallax $\pi_{\odot}$. It is the angle under which the Earth's radius would appear from the Sun:

$$\sin \pi_{\odot} = \frac{R_{\oplus}}{\mathrm{AU}} \tag{13}$$

For the small angle:

$$\mathrm{AU} \simeq \frac{R_{\oplus}}{\pi_{\odot}} \tag{14}$$

Newcomb discussed several determination methods for the solar parallax, among others lunar motion and heliometer measurements of minor planets. Thus the absolute length a is the result of a measurement and model chain: angular observations $\rightarrow$ relative orbit $\rightarrow$ solar parallax / AU $\rightarrow$ a in units of length.

27.3.5.3 e: Eccentricity The eccentricity describes the shape of the ideal Kepler ellipse:

$$e = \frac{r_{\mathrm{aph}} - r_{\mathrm{peri}}}{r_{\mathrm{aph}} + r_{\mathrm{peri}}} \tag{15}$$

$$e \approx 0.2056 \tag{16}$$

Historically, r_{peri} and r_{aph} were not read directly instrumentally as distances. e was determined by fitting the orbital shape to many positional observations. The eccentricity is thus a derived orbital element parameter.

27.3.5.4 T: Sidereal Orbital Period Mercury's sidereal orbital period is approximately

$$T \approx 87.969\,\mathrm{d} \tag{17}$$

It is related to the mean motion n by:

$$n = \frac{2\pi}{T} \tag{18}$$

The mean motion can be determined very precisely from long-term repetitions of Mercury's position. Nevertheless, T is also not a stopwatch measurement of a single isolated revolution but an orbital parameter derived from a long series of positions and periods.

27.3.5.5 c**: Speed of Light** The speed of light is of a different nature compared to a, e, T. It is not obtained from Mercury's orbit. At Einstein's time, terrestrial measurements of the speed of light were already available. Thus c is, with regard to the Mercury orbit problem, a largely external experimental input quantity.

27.3.5.6 $GM_\odot$**: Solar Gravitational Parameter** A planetary orbit does not determine G and $M_\odot$ separately but primarily the product

$$\mu_\odot = GM_\odot \tag{19}$$

In leading Kepler order:

$$T^2 = \frac{4\pi^2 a^3}{GM_\odot} \tag{20}$$

$$GM_\odot = \frac{4\pi^2 a^3}{T^2} \tag{21}$$

This is particularly important for Einstein's historical calculation.

27.3.6 Einstein's 1915 Numerical Form Used

The now widespread presentation is

$$\Delta\varpi = \frac{6\pi GM_\odot}{c^2 a(1-e^2)} \tag{22}$$

$$GM_\odot = \frac{4\pi^2 a^3}{T^2} \tag{23}$$

$$\Delta\varpi = \frac{6\pi}{c^2 a(1-e^2)} \frac{4\pi^2 a^3}{T^2} \tag{24}$$

$$\Delta\varpi = \frac{24\pi^3 a^2}{T^2 c^2 (1-e^2)} \tag{25}$$

This form makes the empirical input quantities immediately visible: a, e, T, c.

Einstein wrote in 1915 that his new theory quantitatively explains the secular Mercury excess of approximately $45''$ per century discovered by Le Verrier. The now widespread pairing "$42.98''$ predicted versus $43.00''$ measured" is therefore not the form in which the historical comparison of 1915 was itself presented.

27.3.7 Where Do the Two Sides of the Historical Comparison Come From?

The genealogy can be represented as two branches of the same astronomical observation system.

Branch 1: Input quantities of the GTR calculation

$$\begin{aligned} \text{astronomical position and time observations} &\Longrightarrow \text{orbit adjustment and astronomical scale} \Longrightarrow a, e, T \\ &\Longrightarrow \Delta\varpi_{\text{GTR}} = \frac{24\pi^3 a^2}{T^2 c^2 (1-e^2)}. \end{aligned}$$

Branch 2: Historical residual

$$\begin{aligned} \text{astronomical position and transit observations} &\Longrightarrow \text{orbital elements + planetary masses + precession + perturbation the} \\ &\Longrightarrow \text{calculated Newtonian Mercury motion} \Longrightarrow \text{observation - calculation} \\ &\Longrightarrow \Delta\dot{\varpi}_{\text{residual}} \sim 43''/\text{Jh}. \end{aligned}$$

Both end quantities thus do not originate from completely separate observation worlds. They draw on the same astronomical survey of the inner solar system developed over decades but use this information in different mathematical combinations.

27.3.8 How Sensitive Is the GTR Number to a, e, T?

$$\Delta\varpi = \frac{24\pi^3 a^2}{T^2 c^2 (1-e^2)} \tag{26}$$

yields logarithmic differentiation

$$\frac{\delta(\Delta\varpi)}{\Delta\varpi} = 2\frac{\delta a}{a} - 2\frac{\delta T}{T} + \frac{2e}{1-e^2}\delta e - 2\frac{\delta c}{c} \tag{27}$$

For Mercury:

$$\frac{2e}{1-e^2} \approx 0.429 \tag{28}$$

This allows one to see transparently which parameter errors enter the theoretical number and how strongly. The eccentricity, for example, is in this sense not a highly effective free lever:

$$43'' \times 0.429 \times 10^{-3} \approx 0.018''/\text{Jh}. \tag{29}$$

The exciting historical question therefore lies less in whether a small shift of e could produce the entire effect, but in the manner in which the comparison value itself was extracted from a coupled long-term adjustment system.

27.3.9 The Number 42.98″: The Theoretical Number Also Has a History

The now precisely cited leading GTR precession amounts, with modern constants, to

$$42.98''/\text{Julian century} \tag{30}$$

Nobili and Will pointed out in 1986 that over decades a slightly different value, approximately 43.03″, was frequently cited. The cause was historically used astronomical constants. With the then best constants, they obtained 42.98″. Thus even the theoretical decimal number is not a timeless numerical output. Its functional form remains the same; its numerical value follows the astronomical constants used.

27.3.10 From Classical Astrometry to Radar and Spaceflight

27.3.10.1 Radar Since the 1960s, direct transit-time measurements of electromagnetic signals to Mercury were added. Thereby the Earth–Mercury distance became accessible much more precisely than with purely optical astrometry. The primary measured quantity is now a signal transit time

$$\Delta t_{\text{signal}} \tag{31}$$

from which, after accounting for propagation and station models, a range quantity is derived. Here too, however, $\dot{\varpi} = 43''/\text{Jh.}$ is not measured directly.

27.3.10.2 MESSENGER With MESSENGER, highly precise radiometric tracking data were added:

- range,
- Doppler,
- spacecraft orbit,
- from this, improved Earth–Mercury geometry.

The data are fed into a numerical model of the solar system. The Mercury ephemeris is thus the result of a global dynamics and parameter estimation.

27.3.11 How a Modern Ephemeris Is Constructed

A modern planetary ephemeris integrates the motion of many bodies simultaneously. Schematically:

$$\ddot{\mathbf{r}}_i = \mathbf{a}_i^{\text{Newton,N–body}} + \mathbf{a}_i^{\text{PPN}} + \mathbf{a}_i^{J_2} + \mathbf{a}_i^{\text{asteroids}} + \cdots \tag{32}$$

The parameters can include:

$$\mathbf{p} = (\mathbf{r}_i(t_0), \mathbf{v}_i(t_0), GM_i, GM_{\text{asteroids}}, J_{2\odot}, \beta, \gamma, \ldots) \tag{33}$$

The numerical integration produces predictions for the actually observed quantities:

$$C_k(\mathbf{p}) = \text{calculated range/Doppler/angular observation} \tag{34}$$

The parameters are determined by minimizing the residuals:

$$\chi^2(\mathbf{p}) = \sum_k \frac{[O_k - C_k(\mathbf{p})]^2}{\sigma_k^2} \tag{35}$$

Thus the modern perihelion precession is not a primary observational quantity but a property of the dynamics reconstructed from all data.

27.3.12 PPN Parameters, J_2 and Correlations

In the parameterized post-Newtonian formalism, the leading relativistic perihelion term depends on

$$\dot{\varpi} = \dot{\varpi}_{\text{PPN}}(\beta, \gamma) + \dot{\varpi}_{J_2} + \dot{\varpi}_{\text{further}} \tag{36}$$

MESSENGER analyses show that in particular β and $J_{2\odot}$ can be strongly correlated. Modern works therefore use additional assumptions or independent information to separate these parameters from each other. A MESSENGER evaluation reported, under the assumption of a metric gravitational theory,

$$\beta = 1 + (-1.6 \pm 1.8) \times 10^{-5} \tag{37}$$

$$J_{2\odot} = (2.246 \pm 0.022) \times 10^{-7} \tag{38}$$

The decisive methodical point lies in the joint parameter estimation: A measured range or Doppler signal generally does not carry the label "this portion is β," "this portion is J_2." The separation occurs via the different temporal and geometric signatures in the overall model.

27.3.13 Standard Ephemeris and Gravitation Test Are Not the Same

In standard ephemerides, PPN parameters can be set to their GTR values:

$$\beta = 1, \qquad \gamma = 1 \tag{39}$$

Such an ephemeris then serves the best possible position prediction under the chosen reference model. For an actual gravitation test, by contrast, special solutions are computed in which the parameters to be investigated are varied or estimated. Modern works then do not simply compare "42.98 versus 43" but investigate how strongly alternative parameter values degrade the entire set of range, Doppler, and astrometric data.

27.3.14 The Modern Significance of the Analytical 42.98″ Formula

$$\Delta\varpi = \frac{6\pi G M_\odot}{c^2 a(1-e^2)} \tag{40}$$

remains didactically and physically important: It isolates the leading relativistic central-field term. The modern precision model of Mercury, however, is more extensive. It includes among other things:

- Newtonian many-body dynamics,
- post-Newtonian many-body terms,
- solar multipoles,
- solar rotation or gravitomagnetic contributions,
- asteroids,
- spacecraft and station models,
- light-time and Doppler models.

Clifford Will also showed in 2018 additional relativistic many-body contributions to Mercury's perihelion motion arising from cross terms between solar and planetary effects as well as gravitomagnetic contributions of moving planets. Thus the isolated 42.98″ number is the leading Schwarzschild-like term, not the complete modern relativistic dynamics of the real solar system.

27.3.15 The Complete Historical and Modern Chain

Historical observation side

$$\begin{aligned}\text{telescope / transit / meridian observation} &\Longrightarrow \text{star catalogs, time, refraction, precession, instrument corrections}\\ &\Longrightarrow \text{orbital element and constants adjustment} \Longrightarrow \text{Newtonian many-body pert}\\ &\Longrightarrow \text{additional secular Mercury term} \sim 43''/\text{Jh.}\end{aligned}$$

Historical GTR side

$$\begin{aligned}\text{GTR derivation} &\Longrightarrow \Delta\varpi = \frac{24\pi^3 a^2}{T^2 c^2(1-e^2)}\\ &\Longrightarrow \text{classically astronomically determined } a, e, T + \text{externally determined } c\\ &\Longrightarrow \sim 43''/\text{Jh.}\end{aligned}$$

Modern observation side

$$\begin{aligned}\text{radar} + \text{spacecraft range} + \text{Doppler} + \text{astrometry} &\Longrightarrow \text{measurement model} + \text{spacecraft orbit}\\ &\Longrightarrow \text{global numerical ephemeris} \Longrightarrow \text{joint parameter estimation}\\ &\Longrightarrow \text{limits for PPN parameters and additional precessions.}\end{aligned}$$

27.3.16 Didactic Balance: What Is Really "Measured"?

Quantity	Meaning	How obtained?
a	semi-major axis	orbit fitting; historically absolute scale via AU/solar parallax
e	eccentricity	fitting of orbital shape to positional observations
T	sidereal orbital period	long-term periodic positional motion / mean motion
c	speed of light	terrestrial or independent physical measurement
$GM_\odot$	solar gravitational parameter	planetary dynamics, equivalently leading from a^3/T^2
ϖ	perihelion longitude	reconstructed from orbital elements
$\dot{\varpi}$	secular change	long-term adjustment of many observations
historical $43''$	unexplained additional term	observation minus classical astronomical model
modern range	radio transit-time distance	direct radiometric observable after measurement model
modern Doppler quantity	relative velocity information	frequency comparison, then model reduction
β, γ, J_2	model parameters	joint global parameter estimation / special test solutions

27.3.17 Conclusion: The Story Behind an Apparently Simple Number

The Mercury test is interesting in the history of science precisely because the short form "$43''$ predicted, $43''$ measured" conceals a much more extensive structure. Historically, at the beginning stood a real astronomical problem known decades before the GTR. The comparison value, however, did not arise as an isolated angle at the eyepiece but as the residual of a large celestial-mechanical adjustment problem. Newcomb's work explicitly shows how orbital elements, secular variations, planetary masses, precession quantities, and fundamental astronomical constants were treated in a coupled manner. On the relativistic side, there is likewise no theory-free decimal number. Einstein's formula requires empirically determined quantities of Mercury's orbit. In its form, which is particularly transparent for the historical calculation, these are a, e, T, and c. The first three come from the same astronomical surveying tradition from which the historical perihelion residual also emerged; c is added as an external physical quantity.

Today the measurement methodology has changed completely, but the basic structure of the inference has not: Primary observables such as range and Doppler are fed into a comprehensive dynamic model. From a global parameter system, ephemerides and limits for relativistic parameters arise. The analytical number $42.98''$ remains the leading, intuitive central-field term, while the actual modern solar system dynamics contains considerably more contributions. Whoever wishes to assess the perihelion precession of Mercury should therefore always distinguish between three things:

$$\text{primary observation} \neq \text{derived orbital parameter} \neq \text{theoretical comparison term} \tag{41}$$

Only this separation makes visible what the famous number actually states and by which historical and mathematical path it arises.

27.3.18 Sources and Further Literature

[1] S. Newcomb, The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy, U.S. Government Printing Office, Washington, 1895. https://commons.wikimedia.org/wiki/File:The_elements_of_the_four_inner_planets_and_the_fundamental_constants_of_astronomy_(IA_elementsoffourin00newcrich).pdf

[2] A. Einstein, Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 1915, S. 831–839. CERN Document Server: https://cds.cern.ch/record/632319

[3] M. Janssen, J. Renn, Einstein and the Perihelion Motion of Mercury, 2021. https://arxiv.org/abs/2111.11238

[4] A. M. Nobili, C. M. Will, The real value of Mercury's perihelion advance, Nature 320, 39–41 (1986). https://doi.org/10.1038/320039a0

[5] C. M. Will, New General Relativistic Contribution to Mercury's Perihelion Advance, Phys. Rev. Lett. 120, 191101 (2018). https://doi.org/10.1103/PhysRevLett.120.191101

[6] A. Fienga et al. / INPOP works on planetary ephemerides and PPN tests; see in particular the literature on varying β- and $J_{2\odot}$ solutions.

[7] A. Genova et al., Solar system expansion and strong equivalence principle as seen by the NASA MESSENGER mission, Nature Communications 9, 289 (2018). https://www.nature.com/articles/s41467-017-02558-1

[8] JPL, Documentation of planetary Development Ephemerides, in particular DE421, with description of the post-Newtonian N-body dynamics used and fixed or estimated parameters.

[9] G. M. Clemence, historical reworkings of Mercury's motion and later perihelion determinations on the basis of extended observation series.

27.4 Gravitational Redshift and Clock Experiments in the GTR

Complete derivation with intermediate steps

27.4.1 Starting Point: Einstein Equation and Static Exterior Field

The Einstein field equation without cosmological constant reads

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{1}$$

For the idealized exterior region of a static, spherically symmetric central mass, one sets

$$T_{\mu\nu} = 0. \tag{2}$$

Thus follows

$$R_{\mu\nu} = 0. \tag{3}$$

Under staticity, spherical symmetry, and asymptotic flatness, the Schwarzschild metric results:

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2 \tag{4}$$

with

$$f(r) = 1 - \frac{2GM}{c^2r}. \tag{5}$$

Up to this point, no weak-field expansion has been performed.

27.4.2 Proper Time of a Stationary Clock

A stationary clock remains at fixed spatial coordinates:

$$dr = d\theta = d\phi = 0. \tag{6}$$

For a material observer:

$$ds^2 = -c^2d\tau^2. \tag{7}$$

Thus

$$-c^2d\tau^2 = -f(r)c^2dt^2. \tag{8}$$

After division by $-c^2$:

$$d\tau^2 = f(r)dt^2. \tag{9}$$

For forward-running proper time:

$$\frac{d\tau}{dt} = \sqrt{f(r)} = \sqrt{1 - \frac{2GM}{c^2r}}. \tag{10}$$

For two stationary clocks at r_1 and r_2:

$$\frac{d\tau_1}{dt} = \sqrt{f(r_1)}, \qquad \frac{d\tau_2}{dt} = \sqrt{f(r_2)}. \tag{11}$$

From this:

$$\frac{d\tau_2}{d\tau_1} = \sqrt{\frac{f(r_2)}{f(r_1)}}. \tag{12}$$

27.4.3 Photon Frequency from Four-Vectors

The frequency that an observer with four-velocity u^μ measures for a light ray with wave vector k^μ is

$$\omega = -k_\mu u^\mu. \tag{13}$$

For a stationary observer:

$$u^\mu = \left(\frac{dt}{d\tau}, 0, 0, 0\right) = \left(\frac{1}{\sqrt{f(r)}}, 0, 0, 0\right). \tag{14}$$

Since the Schwarzschild metric is time-independent, it possesses the timelike Killing vector

$$\xi^\mu = (1, 0, 0, 0). \tag{15}$$

Along a null geodesic,

$$k_\mu \xi^\mu = k_t \tag{16}$$

is constant. Set

$$-k_t \equiv E_\gamma = \text{const}. \tag{17}$$

Then:

$$\omega = -k_t u^t = \frac{E_\gamma}{\sqrt{f(r)}}. \tag{18}$$

For emission at r_1 and reception at r_2:

$$\omega_1 = \frac{E_\gamma}{\sqrt{f(r_1)}}, \qquad \omega_2 = \frac{E_\gamma}{\sqrt{f(r_2)}}. \tag{19}$$

The constant photon parameter cancels:

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{f(r_1)}{f(r_2)}}. \tag{20}$$

Since $\nu = \omega/(2\pi)$:

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{1 - \frac{2GM}{c^2 r_1}}{1 - \frac{2GM}{c^2 r_2}}}. \tag{21}$$

For $r_2 > r_1$, $f(r_2) > f(r_1)$, thus

$$\nu_2 < \nu_1 \tag{22}$$

for a photon traveling upward: gravitational redshift.

27.4.4 Weak-Field Expansion Step by Step

Set the Newtonian potential

$$\Phi(r) = -\frac{GM}{r}. \tag{23}$$

Then

$$f(r) = 1 + \frac{2\Phi(r)}{c^2}. \tag{24}$$

For

$$\frac{\Phi}{c^2} \ll 1 \tag{25}$$

we use

$$(1 + x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots . \tag{26}$$

With

$$x = \frac{2\Phi}{c^2} \tag{27}$$

it follows that

$$\sqrt{f(r)} \simeq 1 + \frac{\Phi(r)}{c^2}. \tag{28}$$

For the frequency ratio:

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{f(r_1)}{f(r_2)}} \simeq \frac{1 + \Phi_1/c^2}{1 + \Phi_2/c^2}. \tag{29}$$

With

$$\frac{1}{1+y} \simeq 1 - y \tag{30}$$

one obtains

$$\frac{\nu_2}{\nu_1} \simeq \left(1 + \frac{\Phi_1}{c^2}\right)\left(1 - \frac{\Phi_2}{c^2}\right). \tag{31}$$

Products of second order are discarded:

$$\frac{\nu_2}{\nu_1} \simeq 1 + \frac{\Phi_1 - \Phi_2}{c^2}. \tag{32}$$

Thus

$$\frac{\Delta\nu}{\nu} \simeq \frac{\Phi_1 - \Phi_2}{c^2}. \tag{33}$$

27.4.5 Local Height Form gh/c^2

Near the Earth's surface, set

$$r_1 = R_\oplus, \qquad r_2 = R_\oplus + h, \qquad h \ll R_\oplus. \tag{34}$$

Then

$$\Phi_2 - \Phi_1 = -GM_\oplus\left(\frac{1}{R_\oplus + h} - \frac{1}{R_\oplus}\right). \tag{35}$$

Expand

$$\frac{1}{R_\oplus + h} = \frac{1}{R_\oplus}\frac{1}{1 + h/R_\oplus} \simeq \frac{1}{R_\oplus}\left(1 - \frac{h}{R_\oplus}\right). \tag{36}$$

Thus

$$\Phi_2 - \Phi_1 \simeq \frac{GM_\oplus}{R_\oplus^2}h. \tag{37}$$

With

$$g = \frac{GM_\oplus}{R_\oplus^2} \tag{38}$$

it follows that

$$\Phi_2 - \Phi_1 \simeq gh. \tag{39}$$

Therefore, in magnitude:

$$\frac{\Delta\nu}{\nu} \simeq \frac{gh}{c^2}. \tag{40}$$

27.4.6 Which Steps Are Exact, Which Are Approximated?

Step	Status
Einstein field equation	covariant starting point
Schwarzschild metric	exact within staticity/spherical symmetry/vacuum
$d\tau/dt = \sqrt{f}$	exact for stationary clock
$\omega = -k_\mu u^\mu$	exact
Killing constant k_t	exact for static metric
Frequency ratio with roots	exact within Schwarzschild model
$\Delta\nu/\nu \simeq \Delta\Phi/c^2$	weak-field approximation
$\Delta\Phi \simeq gh$	additional local height approximation

27.4.7 Methodical Chain

$$\begin{aligned} G_{\mu\nu} &\Longrightarrow R_{\mu\nu} = 0 \Longrightarrow \text{Schwarzschild} \Longrightarrow u^\mu_{\text{stat}} \Longrightarrow \omega = -k_\mu u^\mu \Longrightarrow k_t = \text{const} \\ &\Longrightarrow \frac{\nu_2}{\nu_1} = \sqrt{\frac{f_1}{f_2}} \Longrightarrow \text{weak field} \Longrightarrow \frac{\Delta\nu}{\nu} \simeq \frac{\Delta\Phi}{c^2} \Longrightarrow \frac{gh}{c^2}. \end{aligned}$$

27.5 Lense–Thirring and Frame-Dragging Effect in the GTR

Derivation with significantly expanded intermediate steps

27.5.1 Why Schwarzschild Is Not Sufficient

The Schwarzschild solution presupposes a non-rotating central mass. A rotating source possesses an angular momentum J and is only stationary and axisymmetric. The exact vacuum solution is the Kerr geometry.

27.5.2 Covariant Origin

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

In the exterior region:

$$T_{\mu\nu} = 0, \qquad R_{\mu\nu} = 0. \tag{2}$$

In geometric units $G = c = 1$, the Kerr metric in Boyer–Lindquist coordinates reads

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4Mar\sin^2\theta}{\Sigma}\, dt\, d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{2Ma^2 r \sin^2\theta}{\Sigma}\right) \sin^2\theta\, d\phi^2. \end{aligned} \tag{3}$$

with

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2Mr + a^2, \qquad a = \frac{J}{M}. \tag{4}$$

The new term, decisive compared to Schwarzschild, is

$$g_{t\phi} \neq 0. \tag{5}$$

27.5.3 Slow-Rotation and Far-Field Expansion

For

$$r \gg M, \qquad a \ll r \tag{6}$$

we have

$$\Sigma \simeq r^2. \tag{7}$$

The cross term becomes

$$-\frac{4Mar\sin^2\theta}{\Sigma}\,dt\,d\phi \simeq -\frac{4Ma}{r}\sin^2\theta\,dt\,d\phi. \tag{8}$$

Since $Ma = J$:

$$ds^2 \supset -\frac{4J}{r}\sin^2\theta\,dt\,d\phi \tag{9}$$

in geometric units. With G, c reinserted, the cross term is proportional to

$$\frac{GJ}{c^3 r}. \tag{10}$$

This makes visible: The effect vanishes for $J = 0$.

27.5.4 Linearized Gravitomagnetic Representation

In the weak field, one writes

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1. \tag{11}$$

For a slowly rotating source, the metric contains, besides the scalar potential Φ, a vector potential component A_g:

$$ds^2 \simeq -\left(1 + \frac{2\Phi}{c^2}\right)c^2dt^2 - \frac{4}{c}A_g \cdot dx\,dt + \left(1 - \frac{2\Phi}{c^2}\right)dx^2. \tag{12}$$

For a rotating compact source, the leading far field has the form

$$A_g(\mathbf{r}) = \frac{G}{2c}\frac{\mathbf{J}\times\mathbf{r}}{r^3} \tag{13}$$

with this convention choice. The associated gravitomagnetic curl is

$$B_g = \nabla \times A_g. \tag{14}$$

With the well-known dipole identity

$$\nabla \times \left(\frac{\mathbf{J}\times\mathbf{r}}{r^3}\right) = \frac{3\hat{\mathbf{r}}(\mathbf{J}\cdot\hat{\mathbf{r}}) - \mathbf{J}}{r^3} \tag{15}$$

it follows that

$$B_g = \frac{G}{2c}\frac{3\hat{\mathbf{r}}(\mathbf{J}\cdot\hat{\mathbf{r}}) - \mathbf{J}}{r^3}. \tag{16}$$

27.5.5 Gyroscope Precession

The spin of a freely falling ideal gyroscope is parallel transported. In the weak stationary rotation limit, the g_{0i} component leads to a precession equation

$$\frac{d\mathbf{S}}{dt} = \boldsymbol{\Omega}_{\mathrm{LT}} \times \mathbf{S}. \tag{17}$$

The leading Lense–Thirring precession frequency is

$$\mathbf{\Omega}_{\mathrm{LT}} = \frac{G}{c^2 r^3}\left[3(\mathbf{J}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{J}\right]. \tag{18}$$

The structure is immediately recognizable:

$$|\mathbf{\Omega}_{\mathrm{LT}}| \sim \frac{GJ}{c^2 r^3}. \tag{19}$$

27.5.6 Orbital Node Precession

For a test body, the rotation component is treated as a small perturbation of the leading Kepler orbit. The orbital angular momentum vector L precesses around J. After averaging over a Kepler period, one obtains for the ascending node

$$\dot{\Omega}_{\mathrm{LT}} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}}. \tag{20}$$

The characteristic averaging is based on

$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{a^3(1-e^2)^{3/2}}. \tag{21}$$

This relation follows for a Kepler ellipse from

$$r(\phi) = \frac{p}{1+e\cos\phi}, \qquad p = a(1-e^2), \tag{22}$$

and the temporal weighting over one orbital period.

27.5.7 Where Exactly Does the Approximation Lie?

1. Kerr itself is exact within stationarity, axisymmetry, and vacuum.
2. For the closed Lense–Thirring formulas, one expands in

$$\frac{GM}{c^2 r} \ll 1, \qquad \frac{J}{Mcr} \ll 1.$$

3. Only terms linear in J are retained.
4. The orbit is treated as the leading Kepler orbit.
5. The frame-dragging term is placed perturbatively on top.
6. For the node precession, averaging over the orbital period is performed.

27.5.8 Methodical Chain

$$\begin{aligned} G_{\mu\nu} &\Longrightarrow R_{\mu\nu} = 0 \Longrightarrow \text{Kerr} \Longrightarrow g_{t\phi} \neq 0 \\ &\Longrightarrow \text{far field + slow rotation} \Longrightarrow h_{0i} \Longrightarrow A_g, B_g \Longrightarrow \Omega_{\mathrm{LT}}, \end{aligned}$$

and for orbits

$$\text{Kepler orbit as zeroth order} \Longrightarrow \text{perturbation + orbit averaging} \Longrightarrow \dot{\Omega}_{\mathrm{LT}}.$$

27.6 Binary Pulsars in the GTR

From the Field Equation to the Observed Orbital Decay

27.6.1 Why the Problem Is More Difficult Than Schwarzschild

A binary pulsar system is neither static nor spherically symmetric. The two masses move, the spacetime is time-dependent, and radiative energy is carried away. Therefore, no Schwarzschild-like closed two-body solution exists that could be directly inserted into the observation.

27.6.2 Covariant Starting Point

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

For weakly relativistic orbital motion, a post-Newtonian expansion is introduced:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

with order parameter

$$\epsilon \sim \frac{v^2}{c^2} \sim \frac{GM}{rc^2}. \tag{3}$$

Conservative corrections appear at 1PN, 2PN, ...; the leading radiation back-reaction appears at 2.5PN.

27.6.3 Quadrupole Moment of the Source

In the center-of-mass system, the trace-free mass quadrupole tensor is

$$Q_{ij} = \int \rho(\mathbf{x}, t) \left(x_i x_j - \frac{1}{3}\delta_{ij} r^2 \right) d^3x. \tag{4}$$

For two point masses m_1, m_2, the problem can be reduced to the relative vector $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ with reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \tag{5}$$

Then

$$Q_{ij} = \mu \left(r_i r_j - \frac{1}{3}\delta_{ij} r^2 \right). \tag{6}$$

27.6.4 Leading Radiation Power

In the linear far zone, the GTR yields the quadrupole formula

$$P = -\frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle. \tag{7}$$

The three dots denote the third time derivative.

27.6.4.1 Circular Orbit as a Transparent Special Case For a circular orbit in the xy-plane:

$$\mathbf{r}(t) = a(\cos\omega t, \sin\omega t, 0). \tag{8}$$

Then

$$Q_{xx} = \mu a^2\left(\cos^2\omega t - \frac{1}{3}\right), \tag{9}$$

$$Q_{yy} = \mu a^2\left(\sin^2\omega t - \frac{1}{3}\right), \tag{10}$$

$$Q_{xy} = \mu a^2 \sin\omega t\cos\omega t. \tag{11}$$

With

$$\cos^2\omega t = \frac{1}{2}(1 + \cos 2\omega t), \tag{12}$$

$$\sin^2\omega t = \frac{1}{2}(1 - \cos 2\omega t), \tag{13}$$

$$\sin\omega t\cos\omega t = \frac{1}{2}\sin 2\omega t \tag{14}$$

it becomes visible that the quadrupole oscillates with frequency 2ω.

After differentiating three times, each component is proportional to

$$\mu a^2\omega^3. \tag{15}$$

The contraction and temporal averaging yields

$$\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle = 32\mu^2 a^4\omega^6. \tag{16}$$

Thus

$$P = -\frac{32G}{5c^5}\mu^2 a^4 \omega^6. \tag{17}$$

For the leading orbital order, Kepler is used:

$$\omega^2 = \frac{GM}{a^3}, \qquad M = m_1 + m_2. \tag{18}$$

Thus

$$\omega^6 = \frac{G^3 M^3}{a^9}. \tag{19}$$

Therefore

$$P_{\text{circ}} = -\frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{a^5}. \tag{20}$$

27.6.5 Eccentric Orbit

For a Kepler ellipse

$$r(\phi) = \frac{a(1-e^2)}{1+e\cos\phi} \tag{21}$$

the time-dependent quadrupole tensor is computed along the elliptical motion and subsequently averaged over one orbital period. The complete Peters–Mathews averaging yields

$$\left\langle \dot{E} \right\rangle = -\frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{a^5}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}. \tag{22}$$

The factor

$$F(e) = \frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} \tag{23}$$

is the complete eccentricity correction of the leading quadrupole order.

27.6.6 From Energy Loss to Shrinking Orbit

The leading Newtonian binding energy reads

$$E = -\frac{G\mu M}{2a}. \tag{24}$$

Differentiation:

$$\dot{E} = \frac{G\mu M}{2a^2}\dot{a}. \tag{25}$$

Thus

$$\dot{a} = \frac{2a^2}{G\mu M}\dot{E}. \tag{26}$$

Substitution of the quadrupole power:

$$\dot{a} = -\frac{64}{5}\frac{G^3\mu M^2}{c^5 a^3}F(e). \tag{27}$$

27.6.7 From a to the Orbital Period

The leading Kepler relation reads

$$P_b^2 = \frac{4\pi^2 a^3}{GM}. \tag{28}$$

Logarithmically differentiated:

$$2\frac{\dot{P}_b}{P_b} = 3\frac{\dot{a}}{a} \tag{29}$$

for constant masses. Thus

$$\frac{\dot{P}_b}{P_b} = \frac{3}{2}\frac{\dot{a}}{a}. \tag{30}$$

Substitution:

$$\dot{P}_b = -\frac{96}{5}\frac{G^3\mu M^2}{c^5 a^4}P_b F(e). \tag{31}$$

Now a is eliminated with Kepler:

$$a^3 = \frac{GMP_b^2}{4\pi^2}. \tag{32}$$

Thus

$$a^4 = \left(\frac{GMP_b^2}{4\pi^2}\right)^{4/3}. \tag{33}$$

After rearrangement, one obtains

$$\boxed{\dot{P}_b = -\frac{192\pi}{5}\left(\frac{2\pi G\mathcal{M}}{c^3 P_b}\right)^{5/3} F(e)}. \tag{34}$$

with the chirp mass

$$\boxed{\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}}. \tag{35}$$

27.6.8 What Is Actually Observed?

A radio telescope does not measure a $\dot{P}_b$ formula directly. What is measured are arrival times of individual pulse profiles. From the time stamps, a timing model is fitted. This contains among other things

$$P_b,\ e,\ \omega,\ T_0 \tag{36}$$

as well as post-Keplerian parameters.

The observed period change additionally contains kinematic contributions:

$$\dot{P}_b^{\rm obs} = \dot{P}_b^{\rm int} + \dot{P}_b^{\rm gal} + \dot{P}_b^{\rm Shk} + \cdots, \tag{37}$$

where the Shklovskii term arises from the proper motion of the system.

Thus, for the GTR comparison, one must first reconstruct

$$\dot{P}_b^{\rm int} = \dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal} - \dot{P}_b^{\rm Shk} - \cdots \tag{38}$$

27.6.9 Postulates and Approximation Levels

1. post-Newtonian expansion of the two-body dynamics;
2. point-mass model in the leading radiation calculation;
3. Kepler motion as zeroth order;
4. quadrupole formula as leading radiation order;
5. orbit averaging;
6. Newtonian binding energy for converting $\dot{E} \to \dot{a}$;
7. Kepler relation for converting $\dot{a} \to \dot{P}_b$;

8. timing model and correction of kinematic contributions on the measurement side.

27.6.10 Methodical Chain

$G_{\mu\nu} \to$ PN expansion $\to Q_{ij} \to \dddot{Q}_{ij} \to P_{\rm GW} \to \dot{E} \to \dot{a} \to \dot{P}_b \to$ pulse arrival times $\to$ timing fit $\to \dot{P}_b^{\rm int}$.

27.7 Gravitational Waves and LIGO from the Perspective of the GTR Complete Chain from the Field Equation to the Detector Signal

27.7.1 Covariant Starting Point

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

For an analytical wave theory, the metric is expanded around a flat background:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1. \tag{2}$$

The inverse tensor up to first order is

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + O(h^2). \tag{3}$$

27.7.2 First-Order Christoffel Symbols

Since $\eta_{\mu\nu}$ is constant:

$$\partial_\alpha \eta_{\mu\nu} = 0. \tag{4}$$

Thus

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right). \tag{5}$$

Up to $O(h)$:

$$\Gamma^{(1)\rho}_{\mu\nu} = \frac{1}{2} \eta^{\rho\sigma} \left(\partial_\mu h_{\sigma\nu} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu} \right). \tag{6}$$

Products $\Gamma\Gamma$ are already $O(h^2)$ and are discarded in the linearization.

27.7.3 Linear Ricci Tensor

From

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\nu \Gamma^\rho_{\mu\rho} + \Gamma\Gamma - \Gamma\Gamma \tag{7}$$

it follows to first order

$$R^{(1)}_{\mu\nu} = \partial_\rho \Gamma^{(1)\rho}_{\mu\nu} - \partial_\nu \Gamma^{(1)\rho}_{\mu\rho}. \tag{8}$$

After substitution:

$$R^{(1)}_{\mu\nu} = \frac{1}{2}\left(\partial_\rho \partial_\mu h^\rho{}_\nu + \partial_\rho \partial_\nu h^\rho{}_\mu - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h\right). \tag{9}$$

The linear Ricci scalar is

$$R^{(1)} = \partial_\mu \partial_\nu h^{\mu\nu} - \Box h. \tag{10}$$

27.7.4 Trace Reversal and Lorenz Gauge

Define

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h. \tag{11}$$

Then

$$\bar{h} = -h. \tag{12}$$

The Lorenz gauge condition is

$$\partial^\mu \bar{h}_{\mu\nu} = 0. \tag{13}$$

Under this condition, the linear Einstein tensor reduces to

$$G^{(1)}_{\mu\nu} = -\frac{1}{2}\Box \bar{h}_{\mu\nu}. \tag{14}$$

Thus the Einstein equation becomes

$$-\frac{1}{2}\Box \bar{h}_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{15}$$

so

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}. \tag{16}$$

In vacuum:

$$\Box \bar{h}_{\mu\nu} = 0. \tag{17}$$

27.7.5 Plane Wave

Set

$$\bar{h}_{\mu\nu} = A_{\mu\nu} e^{ik_\alpha x^\alpha}. \tag{18}$$

Then

$$\Box \bar{h}_{\mu\nu} = -k^\alpha k_\alpha A_{\mu\nu} e^{ikx}. \tag{19}$$

The vacuum equation demands

$$k^\alpha k_\alpha = 0. \tag{20}$$

The wave thus propagates on the Minkowski background with c.

The Lorenz condition yields

$$k^\mu A_{\mu\nu} = 0. \tag{21}$$

27.7.6 TT Gauge and Polarizations

For a wave in the z-direction, the remaining gauge freedom allows the transverse-traceless form to be chosen:

$$h^{\mathrm{TT}}_{0\mu} = 0, \qquad h^i{}_i = 0, \qquad \partial_i h_{ij} = 0. \tag{22}$$

Then remains

$$h^{\mathrm{TT}}_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{23}$$

27.7.7 Why an Interferometer Measures Something: Geodesic Deviation

The coordinate-invariant relative acceleration of neighboring freely falling test masses is described by

$$\frac{D^2\xi^\mu}{D\tau^2} = -R^\mu{}_{\alpha\nu\beta}u^\alpha\xi^\nu u^\beta \tag{24}$$

For slowly moving test masses and a TT wave:

$$\frac{d^2\xi^i}{dt^2} = -R^i{}_{0j0}\xi^j. \tag{25}$$

To linear order:

$$R_{0i0j} = -\frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}}. \tag{26}$$

Thus

$$\ddot{\xi}^i = \frac{1}{2}\ddot{h}_{ij}^{\mathrm{TT}}\xi^j. \tag{27}$$

For a pure +-polarization and an arm in the x-direction:

$$\frac{\Delta L_x}{L} \simeq +\frac{1}{2}h_+. \tag{28}$$

For the orthogonal y-arm:

$$\frac{\Delta L_y}{L} \simeq -\frac{1}{2}h_+. \tag{29}$$

Thus the differential arm change is

$$\frac{\Delta L_x - \Delta L_y}{L} \simeq h_+. \tag{30}$$

27.7.8 Source Field in the Far Zone

The retarded solution of the linearized equation is schematically

$$\bar{h}_{\mu\nu}(t,\mathbf{x}) = \frac{4G}{c^4}\int \frac{T_{\mu\nu}\left(t - |\mathbf{x}-\mathbf{x}'|/c, \mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|}\, d^3x'. \tag{31}$$

In the far field R of a slow compact source, the multipole expansion leads to the quadrupole term:

$$h_{ij}^{\mathrm{TT}}(t,\mathbf{x}) = \frac{2G}{c^4R}\ddot{Q}_{ij}^{\mathrm{TT}}\left(t - \frac{R}{c}\right). \tag{32}$$

27.7.9 Circular Binary System and Chirp

For two bodies with total mass M, reduced mass μ, and orbital frequency ω:

$$\omega^2 = \frac{GM}{a^3} \tag{33}$$

to leading order.

The radiated power is

$$P = \frac{32}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{a^5}. \tag{34}$$

The binding energy:

$$E = -\frac{G\mu M}{2a}. \tag{35}$$

Energy balance:

$$\dot{E} = -P. \tag{36}$$

From this follows

$$\dot{a} = -\frac{64}{5}\frac{G^3 \mu M^2}{c^5 a^3}. \tag{37}$$

Since

$$\omega = (GM/a^3)^{1/2}, \tag{38}$$

we have

$$\frac{\dot{\omega}}{\omega} = -\frac{3}{2}\frac{\dot{a}}{a}. \tag{39}$$

Substituting $\dot{a}$:

$$\dot{\omega} = \frac{96}{5}\frac{G^{5/3}}{c^5}\mathcal{M}^{5/3}\omega^{11/3}. \tag{40}$$

With GW frequency

$$f = \frac{\omega}{\pi} \tag{41}$$

it follows that

$$\boxed{\dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}}. \tag{42}$$

27.7.10 From Signal to Parameter Fit

The detector measures

$$s(t) = n(t) + h(t;\theta), \tag{43}$$

where θ contains parameters such as masses, spins, distance, orientation, and coalescence time.

In the frequency domain, a noise-weighted scalar product is used:

$$(a|b) = 4\,\Re \int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}\,df. \tag{44}$$

The matched-filter signal strength is schematically

$$\rho = \frac{(s|h)}{\sqrt{(h|h)}}. \tag{45}$$

For real mergers, the templates originate from a combination of

$$\text{PN inspiral} + \text{numerical relativity} + \text{ringdown perturbation theory}. \tag{46}$$

27.7.11 Approximation and Model Levels

1. Background decomposition $g = \eta + h$;
2. Truncation of all $O(h^2)$ terms in the analytical wave theory;
3. Lorenz gauge;
4. TT gauge for the far-field wave;
5. Slow source and multipole expansion for quadrupole formula;
6. Kepler/PN structure for inspiral;
7. Numerical full GTR for strong merger;

8. Perturbation theory for ringdown;

9. Detector response and statistical template evaluation.

27.7.12 Methodical Chain

$G_{\mu\nu} \to g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \to G^{(1)}_{\mu\nu} \to \Box\bar{h}_{\mu\nu} \to \text{TT wave} \to R_{0i0j} \to \Delta L/L \to Q_{ij} \to h(t) \to \text{PN + NR + ringdown} \to \text{templa}$

28 Literature

References

[1] Tiesinga, E., Mohr, P. J., Newell, D. B., & Taylor, B. N. (2025). CODATA Recommended Values of the Fundamental Physical Constants: 2022. *Reviews of Modern Physics*.

[2] Abbott, B. P. et al. (LIGO Scientific Collaboration and Virgo Collaboration). (2016). Observation of Gravitational Waves from a Binary Black Hole Merger. *Physical Review Letters*, 116, 061102.

[3] NIST. (2026). CODATA Value: Newtonian Constant of Gravitation. `https://physics.nist.gov/cgi-bin/cuu/Value?bg`

[4] Riess, A. G. et al. (2025). A Comprehensive Measurement of the Local Value of the Hubble Constant with 2.4% Uncertainty. *The Astrophysical Journal*.

[5] Abbott, B. P. et al. (2017). GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. *Physical Review Letters*, 119, 161101.

[6] Wald, R. M. (1984). *General Relativity*. University of Chicago Press.

[7] Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1973). *Gravitation*. W. H. Freeman.

[8] Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*, 49, 769–822.

[9] Minkowski, H. (1908). Raum und Zeit. *Physikalische Zeitschrift*, 10, 104–111.

[10] Weinstein, G. (2016). Einstein and Gravitational Waves 1936–1938. *arXiv:1602.04025*.

[11] Planck Collaboration. (2018). Planck 2018 Results. *Astronomy & Astrophysics*, 641, A6.

[12] Arp, H. C. (1997). *Seeing Red: Redshifts, Cosmology and Academic Science*. Apeiron.

[13] Unzicker, A. (2020). Gravitationswellen: Stilles Fiasko. `https://www.telepolis.de/article/Fake-News-aus-dem-Universum-4442282.html`

[14] Freyling, D. (2026). Elementarkörpertheorie (EKT). `http://www.kinkynature.com/ektheorie/`

[15] Johnson, C. (2016). The Absurdity of Modern Physics: LIGO. `https://claesjohnson.blogspot.com/2016/02/absurdity-of-modern-physics-ligo.html`

[16] Fontane, T. (1890). *Die Poggenpuhls*. (Quoted from: "Wir stecken bereits tief in der Dekadenz. Das Sensationelle gilt und nur einem strömt die Menge noch begeisterter zu, dem baren Unsinn.")

28.1 Source Directory

References

[1] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, FLAG Review (2016), arXiv:1607.00299.

[2] J. Finkenrath and A. Jüttner, *Strategy for the Future of Lattice QCD*, (March 2025).

[3] S. Aoki et al., *Physical point simulation in 2+1 flavor lattice QCD*, Phys. Rev. D81 (2010) 074503. arXiv:0911.2561

[4] S. Aoki et al., *Non-perturbative renormalization of quark mass in Nf = 2 + 1 QCD*, JHEP 1008 (2010) 101. arXiv:1006.1164

[5] S. Dürr et al., *Lattice QCD at the physical point: light quark masses*, Phys. Lett. B701 (2011) 265. arXiv:1011.2403

[6] S. Dürr et al., *Lattice QCD at the physical point: simulation and analysis details*, JHEP 1108 (2011) 148. arXiv:1011.2711

[7] R. Arthur et al., *Domain wall QCD with near-physical pions*, Phys. Rev. D87 (2013) 094514. arXiv:1208.4412

[8] N. Carrasco et al., *Up, down, strange and charm quark masses with Nf = 2+1+1 twisted mass lattice QCD*, Nucl. Phys. B887 (2014) 19. arXiv:1403.4504

[9] A. Bazavov et al., *Charmed and light pseudoscalar meson decay constants*, Phys. Rev. D90 (2014) 074509. arXiv:1407.3772

[10] B. Chakraborty et al., *High-precision quark masses and QCD coupling from nf = 4 lattice QCD*, Phys. Rev. D91 (2015) 054508. arXiv:1408.4169

[11] T. Blum et al., *Domain wall QCD with physical quark masses*, Phys. Rev. D93 (2016) 074505. arXiv:1411.7017

[12] M. Gell-Mann, R. J. Oakes and B. Renner, *Behavior of current divergences under SU(3)xSU(3)*, Phys. Rev. 175 (1968) 2195.

[13] B. Bloch-Devaux, *Results from NA48/2 on pi-pi scattering lengths*, PoS CONFINEMENT8 (2008) 029.

[14] J. Gasser, A. Rusetsky and I. Scimemi, *Electromagnetic corrections in hadronic processes*, Eur. Phys. J. C32 (2003) 97. arXiv:hep-ph/0305260

[15] A. Rusetsky, *Isospin symmetry breaking*, PoS CD09 (2009) 071. arXiv:0910.5151

[16] J. Gasser, *Theoretical progress on cusp effect and Kl4 decays*, PoS KAON07 (2008) 033. arXiv:0710.3048